Exchange Rates
with Substitutable Currencies

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INTRODUCTION

Economists, analysts, and forecasters have used many different models to explain the movements of the Canadian dollar, as well as predict its future course. The many different approaches, including current account, monetary, interest-rate-differential, and momentum models have been tried, but none has yet proven totally satisfactory. The approach in this chapter will not fill this void, but it is designed to add to the literature and to attempt to move in a new direction. The following analysis is derived from the currency substitution literature first discussed by Boyer. From this literature one knows that for particular countries, especially Canada, demand-side substitution is important in the determination of capital flows and exchange rate movements. New understanding of how a monetary authority should implement policy to meet stated goals is also implied in the work.

To understand why an exchange rate has moved, much less to forecast it, one must understand the behavior of the market forces driving it. The next section describes three such possible models, all within the context of

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A general monetary model, which could be used to describe the Canadian dollar market. The third section presents estimates of two of the models’ reduced forms. Finally, conclusions are drawn in the last section.

**GENERAL MONETARY MODELS**

The basic monetary model, commonly called the monetary approach to exchange rate analysis, is a subset of the broader set of monetary models of exchange rate determination. We begin with the basic model. Following Humphrey and Lawler, this model assumes that all excess monetary pressures lead to adjustments both in domestic price inflation and in the foreign exchange markets, by a depreciation or appreciation of currency. The model implicitly assumes that currencies are not substitutable in the portfolios of domestic residents or in the portfolios of third parties.

The basic model is outlined in seven equations:

\[
\begin{align*}
MC^d &= MC^s \quad (1) \\
MUS^d &= MUS^s \quad (2) \\
M^d &= yF_{t-1} \quad (3) \\
M^* &= m \quad (4)
\end{align*}
\]

where \( C \) = Canada; \( M^d \) = money demand; \( M^s \) = money supply; \( m \) = the stock of money created by the monetary authorities; \( i \) = short-term interest rate; \( y \) = real income; \( P \) = price level; and \( NA \) = North America, which is the combination of a Canadian parameter, such as money supply, and the same U.S. parameter. Combining 1 through 4, rearranging, and taking the first difference of the log values yields

\[
\hat{P} = M - Y + \hat{t}
\]

(5)

where \( \hat{X} \) = for all variables means percentage change of \( X \), and \( \hat{a} \) = differential of \( X \)

Or,

\[
\begin{align*}
\hat{MC} &= MC - YC + \hat{dC} \quad (5a) \\
\hat{PUS} &= MUS - \hat{YUS} + \hat{dUS} \quad (5b)
\end{align*}
\]
Then, assuming purchasing-power parity, this yields
\[ E = PC - PUS \]  \hspace{1cm} (6)

where \( E \) = the exchange rate in Canadian (C) dollar/U.S. dollar

Finally, combining terms à la Humphrey and Lawler yields
\[ \dot{E} = -(\gamma C - \gamma YUS) + (\dot{MC} - \dot{MUS}) + \delta(\dot{C} - \dot{US}) \]  \hspace{1cm} (7)

Alternatively, one may assume that there exists perfect currency substitution. In this case, whether induced from the demand side or the supply side, there would be no change in the rate of exchange between the two currencies.

Following equation 6 then,
\[ E = 0 \]  \hspace{1cm} (8)

and, since 6 holds in the long run,
\[ PC - PUS \]  \hspace{1cm} (9)

Moreover, under perfect currency substitution on the demand side, a price determination model—where the appropriate money supply is the total money supply in North America—can be derived:
\[ PUS = M\hspace{1cm}S\hspace{1cm}NA - (\gamma C + \gamma YUS) + \delta(\dot{US}) \]  \hspace{1cm} (10)

where \( \delta(\dot{US}) = \delta(\dot{IC}) \), and
\[ M\hspace{1cm}S\hspace{1cm}NA = MUS + MC \]

There would be no need for currency realignment. Capital would flow into people's portfolios as needed, and quantity adjustments would be sufficient to offset the need for price (exchange rate) adjustments.

Inflation in Canada would in the long run be equal to that in the United States. Since real output would be different, either the rate of increase in domestic money or the velocity of circulation would be different. Or, in a world of perfect demand-side substitutability, the relevant money for price determination should be only North American money. Money would be some linear combination of the supply of money in both the United States and Canada. Thus, the relevant rate of growth in money would be the rate of growth in both U.S. and Canadian money, and
the amount of growth which could be supported in each country would depend upon the movement of velocity.

One interesting point in this example—similar to the relationship of Texas to New York—is that the amount of money from the larger North American pool needed to support real activity in one part of the continent would be supplied as needed. In this example, Canada is one part of a larger capital market from which it draws the set of funds needed to support a prescribed level of economic activity. If Canadian activity is substantially higher than that in the United States, funds will flow into Canada from the United States. Thus, the rate of growth of C dollars in the system is of no importance to the rate of economic expansion (or contraction) of Canada or the rate of inflation. From a measurement perspective, it should be noted that if the Canadian economy was clearly growing at a longer-term rate higher than that of the United States, then this would merely mean that the velocity of North American money in Canada would, ex post, grow faster than that in the United States.

The fact remains, however, that the two currencies are not perfectly substitutable; they do move against each other. But the literature does tell us that a model which takes currency substitution into consideration is useful in understanding the monetary relationships in Canada and between Canada and the United States. The third model is of this type; it is first presented in structural form, and eventually a reduced form which can be estimated is derived.*

Following equation 10 and assuming nonperfect currency substitution,

\[ PC = PUS + E + \phi \]  (11)

where \( \phi \) represents the scope for currency substitution. Depending on whether \( \phi \) moves toward greater substitutability or lesser substitutability, the currency portfolio effect will tend to cause the C dollar to vary more or less than would be suggested by a model with no currency substitution.

The parameter \( \phi \) can be defined as:

\[ \phi = \frac{\text{MC}}{\text{MSNA}} [A - (\text{MC}^* - \text{MUS}^*)] \]  (12)

where \( \text{MC}^* \) = the expected X, and A is a constant

*The general approach that follows is outlined in the sources listed in reference note 6, but differs in that it simplifies the methodology by not utilizing the PPP equation in a more transformation equation to obtain the underlying monetary variables.
The scope for currency substitution will change if expected monetary policy in Canada changes. The scale variable (MC/MSNA) is a measure of the relative size of the Canadian money stock to total North American money.

It should be noted that \( (MC^* - MUS^*) \) is likely to be zero most of the time. Only when political events or natural circumstances change the long-run expected monetary policy will \( (MC^* - MUS^*) \) not be zero. Still, rewriting 12

\[
\dot{\phi} = MC - M\text{SNA} - \left( \frac{G}{A - G} \right) \dot{\hat{G}} \tag{12a}
\]

where \( G = (MC^* - MUS^*) \)

Rearranging 11 and combining 12a yields

\[
\dot{\hat{E}} = PC - \rho \hat{US} - \dot{\hat{MC}} + M\text{SNA} + \left( \frac{G}{A - G} \right) \dot{\hat{G}} \tag{13}
\]

The key to this model is the price-determination equation 11 and the exchange rate determination equation 13. These differ from the standard model in several ways. In the long run, prices are expected to move in roughly the same pattern, compensated for by the scope of currency substitution and the movement in the exchange rate. And, in general, the exchange rate is not expected to move enough to realign portfolios unless there is a major shift in policy. The expected short-run price and exchange rate movements, however, may vary from this long-run trend. Furthermore, the money supplies of both Canada and the United States remain at the equation even if, ex post, \( G \) does not change. This results from the fact that the existence of the two currencies in residents' portfolios is a sufficient condition to make the two currencies important determinants of the exchange rate (note, however, that they can be offsetting).

In a simplistic fashion the model may be thought of as an embellished purchasing-power parity (PPP) model of exchange rate determination. But then, this is what most monetary models of exchange rate determination actually are. In this model the embellishments are derived from a view of the world that allows for currency substitution. In the standard monetary model, the embellishment is the use of PPP to make the transformation from price determination to exchange rate determination. In both cases PPP is the bridge between a long-run logical implication of price arbitrage to a more fundamental model of the process that allows prices to differ if the exogenous variables are monetary.
In an effort to highlight the notion that all money in North America is substitutable, we have not considered the fact that both the C dollar and the U.S. dollar are held by third parties as well. These third parties also hold several currencies—the majority of which will not be either U.S. or Canadian dollars—just as Americans and Canadians may hold currencies other than dollars. The model above does not totally take this into account.

In an attempt to consider this fact, we can refer to equation 12 and open North America to yield

$$\phi = \frac{MC}{M_{SNA}} (A - G)[X]$$

where \(X\) = an exchange rate measure

Individuals may hold DMs, C dollars, and U.S. dollars. Suppose an exogenous shock affects the DM/U.S. dollar rate. There would be an incipient buying (selling) of U.S. dollars by the third parties. If there is no change in MC, MUS, or G, there could still be an effect on the C dollar/U.S. dollar rate, unless there existed either symmetrical selling of C dollars and U.S. dollars or perfectly matched symmetrical distribution of buying and selling. That is, suppose only U.S. dollars were sold (bought) by the holders of DM; then there would exist an excess supply (demand) of U.S. dollars vis-à-vis C dollars. If C dollar-holders did not hold this extra supply, the C dollar would change in value. Depending upon the sign of \(\phi\) in relation to \(E\), a third-party selling of U.S. dollars could make the C dollars appreciate or depreciate. Though traditional closed-economy models would argue that a decline in the DM/U.S. dollar rate will lead to a decline in the C dollar/U.S. dollar rate as more U.S. dollars are supplied, this may not be the case. These portfolio-holders, seeing that the two currencies are substitutable, may dump equal amounts of C dollars and U.S. dollars, thereby creating an excess supply of C dollars relative to U.S. dollars. The exchange rate could then move simply as a result of the relative size of the two stocks of money. If portfolio holders consider the two substitutable in their own portfolios, this is the likely result.

$$\dot{E} = \dot{PC} - \dot{PUS} - \frac{MC}{M_{SNA}} + \frac{G}{A - G} \dot{G} - \dot{X}$$

In either case it appears likely that as the DM/U.S. dollar exchange rate becomes more uncertain, the C dollar would weaken. That is, on net, if third-party activities heightened instability in the C dollar/U.S. dollar rate, uncertainty over the C dollar/U.S. dollar rate would tend to reduce foreign
holdings of C dollars relative to U.S. dollars simple because of portfolio consideration of relative size. Thus, the relevant variable is the variability of the rates, not the rate changes themselves.

**EMPIRICAL RESULTS**

From equation 7 the reduced-form equation for the standard monetary model is

\[ \hat{E} = B_0 + B_1 (\hat{Y}_C - \hat{Y}_{US}) + B_2 (\hat{M}_C - \hat{M}_{US}) + B_3 (\hat{i}_C - \hat{i}_{US}) + u \]  

where \( u \) is a stochastic disturbance term, 
\( B_2, B_3 < 0, \) and 
\( B_1 > 0 \)  

From equation 14 the reduced-form equation is

\[ \hat{E} = B_0 + B_1 (\hat{P}_C - \hat{P}_{US}) + B_2 \hat{M}_C + B_3 \hat{M}_{US} + B_4 A - G \hat{G} + \nu \]  

where \( \nu \) is a stochastic disturbance term, 
\( B_1, B_2, B_4 > 0, \) and 
\( B_3 < 0 \)  

From equation 15 the reduced-form equation is

\[ \hat{E} = B_0 + B_1 (\hat{P}_C - \hat{P}_{US}) + B_2 \hat{M}_C + B_3 \hat{M}_{US} + B_4 \frac{A - G}{A} \hat{G} + B_5 \hat{X} + w \]  

where \( w \) is a stochastic disturbance term 
\( B_1, B_3, B_5 > 0, \) and 
\( B_2 < 0 \)
The results of the three sets of regressions are consistent with a model of North American exchange rate determination that is based upon the general guidelines of a monetary model, but which, depends upon the consideration of currency substitution. The model that assumes no currency substitution performs poorly by the overall standards of the more general model that considers currency substitution. The F-values, size of the coefficients, and the t-statistics suggest that the proper monetary model of exchange rate determination should be built around the concept of portfolio in North America containing both U.S. dollars and C. dollars.

Specifically, regressions for four time periods are reported. Regressions for the total data period, 1971 first quarter (Q1) to 1981 Q4, are reported for the standard monetary model (Table 16.1) and for the currency substitution model that does not consider foreign holders of U.S. dollars and C. dollars (Table 16.2). Because of data considerations, regressions that consider third parties (Table 16.5) begin in 1973. The fourth quarter of 1979 is considered a breaking point—owing to the significant change in the direction of U.S. monetary policy—even though the short-time period following 1979 Q4 leaves few degrees of freedom.

The variable G is proxied by a dummy variable that considers political influences on the expectation of the future direction of Canadian versus U.S. monetary policy. Specifically, the Quebec election and the National Energy Plan periods are considered. The variable X is proxied by the mean-adjusted variance of the DM/U.S. dollar exchange rate. The term VARDM is calculated from daily observations on 9 quarterly basis. All data are quarterly. Regressions are OLS.

In general, the standard model results are not heartening. The coefficients are typically not significantly different from zero and do not appear very stable over the sample period. Moreover, the signs are wrong on the monetary variables. Of particular interest, however, is the coefficient on the interest-rate term. During 1973 to 1979 it was unimportant in explaining exchange rate movements. This was, in general, a period of interest rate targeting by the authorities, which usually find the central banks following market movements along a general trend toward higher rates in both countries. During the period since 1979, however, it became the most significant variable. During this period interest rates moved around substantially and conveyed a great deal of information about expected inflation and expected policy.

The results for the model with currency substitution are much better. The corrected R²'s are high, the F-values are high, and the coefficients, for the most part, are well behaved. They tend to be significant and have the correct signs. The key variable is the overall stock of money in North America, resulting from the scale term in the scope for the currency-
<table>
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<th>PCTEY</th>
<th>Constant</th>
<th>(YC - YUS)</th>
<th>(MC - MUS)</th>
<th>(6C - 6US)</th>
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Note: The terms used in Tables 16.1, 16.2, and 16.3 are as follows:
PCTEY = Annual percentage change in the spot exchange rate (Canadian dollar per U.S. dollar). Source: International Financial Statistics.
MC = Annual percentage change in Canadian M3 quarterly data. Source: Bank of Canada Review.

* Dummy variables.

VARصاح = Quarterly variance of the DM/U.S. dollar spot bid rate divided by the quarterly mean of the DM/U.S. dollar spot bid rate.
### Table 16.2

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<th>PCTEY</th>
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<th>DW</th>
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substitution variable. This is expected under a model of nonperfect currency substitution. Consideration of the third parties holding dollars, by the inclusion of the X variable, improves the regressions. In this more fully specified version of the model the coefficients are close to their anticipated values, the DW's remain well behaved, the t-statistics are strong, and the theory is supported.

CONCLUSIONS

The conclusions from this modeling exercise are straightforward. First, the standard monetary model of exchange rate determination, which assumes zero currency substitution between U.S. and Canadian dollars, appears incorrect. Second, a model of perfect currency substitution is not appropriate for guiding market participants' behavior, but may be considered by the monetary authorities if the alternative is simply to assume that there is no currency substitution. And third, a simple model which begins with the notion that currency substitution is neither perfect nor zero yields a satisfactory explanation of exchange rate determination in North America.

One other point is worth noting. The models do not depend upon difficult or complicated rationalizations of more basic models, nor do they attempt to embellish the existing monetary literature. Rather, they are designed to demonstrate that a very simple model of exchange rates in North America may be used if it is based upon the correct theory of price and exchange rate determination.

NOTES


2. For a general discussion of the monetarist approach, see Blinder H. Putnam and D. Sykes Wilford, eds. The Monetary Approach to International Adjustment (New York: Praeger, 1978).


5. For the perfect substitution case, see Wilford and Nattress, "Economic Integration of North America: Miles, in "Currency Substitution: Perspectives, Implications and Empirical Evidence," Aten Canadian dam in a production function framework.
