At maturity—the delivery date—the futures contract can be converted directly to the underlying asset. Therefore, at maturity, the price of the futures and the cash price of the asset must be the same. Prior to maturity, the cash price and the futures price need not be the same. But because the futures price must converge to the cash price at maturity, we know that there must exist some systematic relation between the two prices. The relation between cash and futures prices is the subject of this chapter.

**Futures Prices and the Cost of Carry**

To examine the relation between cash and futures prices, let's begin by looking at some illustrative prices. In Figure 7-1 we provide data on the cash prices—the spot prices—of some grains and feed on Thursday, March 3, 1968. Figure 7-2 provides data on corresponding futures prices on the same day.

Look, for instance, at wheat prices. The spot price of no. 2 soft red wheat is $2.935 (293.5 cents) per bushel in St. Louis. But the price on a futures contract on the Chicago Board of Trade that is deliverable in no. 2 soft red wheat and will mature in only thirteen days—on the third Wednesday in March—is 304.5 cents per bushel. Moreover, as Figure 7-2 indicates, the futures price of wheat in May is higher than that in March; the futures price of wheat in July is higher than that in May and so on. Hence, the data on wheat prices demonstrate two important characteristics of futures prices:
1. The futures price of a commodity or asset, \( F \), is greater than the spot price, \( P \).

2. The futures price, \( F \), rises as the time to maturity increases. These characteristics reflect the cost of carry for a futures contract and illustrate a critical arbitrage relation. To see how this works, an example will be most useful.

**Example**

**Cost of carry**

Suppose the spot price of no. 2 red wheat in a Chicago warehouse is 300 cents per bushel, the yield on a one-month T-bill is 6%, and the cost of storing and insuring one bushel of wheat is 4 cents per month. Given these data, what can be said about the price today for a one-month futures contract, that is, the price of a futures contract that has one month to maturity?

Instead of buying a futures contract on wheat, one could buy wheat today and store it for one month. In one month, the total cost of this transaction to the buyer is the cost of using the money for one month (the forgone interest) and the cost of storing and insuring the wheat:

1. In our wheat example, this will be true even after we adjust for the transportation cost between St. Louis and Chicago.
2. We will ignore any seasonality, such as that evident in the futures price of oats.
3. This example is adapted from Sharpe, *op. cit.*
### Figure 7-2. Futures Prices.

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Open High</th>
<th>Open Low</th>
<th>Close High</th>
<th>Close Low</th>
<th>Close</th>
<th>Open Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat</td>
<td>290.50</td>
<td>289.50</td>
<td>289.50</td>
<td>288.50</td>
<td>289.00</td>
<td>-1.75</td>
</tr>
<tr>
<td>Soybeans</td>
<td>12.75</td>
<td>12.25</td>
<td>12.50</td>
<td>12.25</td>
<td>12.40</td>
<td>-0.75</td>
</tr>
<tr>
<td>Corn</td>
<td>5.00</td>
<td>4.75</td>
<td>4.75</td>
<td>4.50</td>
<td>4.60</td>
<td>-0.10</td>
</tr>
</tbody>
</table>


\[ 300.0 + \frac{(38/3600)0.06}{4} = 305.5 \]

Hence, 305.5 cents per bushel is the maximum that should be paid for the one-month futures contract. If the futures contract was priced at 306, a party could sell futures contracts, buy wheat today, store it for one month, and make a riskless profit—an arbitrage profit.

The point of our wheat example is that the futures price must be related to the spot price through the cost of carry, \( c \), for the futures contract in
question. As shown in the preceding example for commodity contracts, arbitrage guarantees that the futures price will be less than or equal to the spot price plus the cost of carry:

\[ F \leq P + c \]  

(7-1)

We saw in the example that if \( F > P + c \), a trader could make a riskless profit by taking a long position in the asset and a short position in the futures contract. If \( F < P + c \), the arbitrage strategy would be to buy the futures and sell the commodity short but short sales of a physical commodity are difficult.4

However, it is with futures on financial assets that we will be concerned and short selling is possible for financial assets. In this case, if \( F < P + c \), a trader can make an arbitrage profit. Hence, with the financial futures, the principle of no arbitrage requires that Equation (7-1) be a strict equality:

\[ F = P + c \]  

(7-2)

In Figure 7-3, we provide data on financial futures for the same date (March 3, 1988) that was used to illustrate futures prices for commodities. We have annotated the data for the Eurodollars futures contracts to show how these data are interpreted.

The cost-of-carry relation holds for financial assets as it does for commodities. The only difference is in the things that make up the cost of carry. For commodities, the cost of carry is simply the cost of storing and insuring the commodity. For financial assets, the cost of carry refers to the net financing costs (coupon income minus financing costs). Put another way, the cost of carry is the difference between the opportunity cost of holding the asset (the short-term interest rate—the financing cost) and the yield earned from holding the financial asset (e.g., the coupon payments received in the case of holding bonds).

Moreover, Equation (7-2) is particularly useful for considering what is referred to as basis in a futures contract. Basis is defined as the difference between the futures price and the spot price:

\[ \text{Basis} = F - P \]  

(7-3)

4. However, in the context of the preceding wheat example, if one had a large enough inventory of wheat, the strategy could be accomplished by reducing inventory (selling wheat on the spot market) and buying futures.
## Financial Futures Prices

### Futures Prices

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Open</th>
<th>High</th>
<th>Low</th>
<th>Close</th>
<th>Last</th>
<th>Bid</th>
<th>Ask</th>
<th>Change</th>
<th>% Change</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. DOLLAR BILL 1 YR</td>
<td>1.2750</td>
<td>1.2775</td>
<td>1.2750</td>
<td>1.2762</td>
<td>1.2762</td>
<td>1.2762</td>
<td>1.2762</td>
<td>-0.0012</td>
<td>-0.97%</td>
<td>3,285,460</td>
</tr>
<tr>
<td>U.S. DOLLAR BILL 2-1/2 YR</td>
<td>1.2900</td>
<td>1.2910</td>
<td>1.2900</td>
<td>1.2905</td>
<td>1.2905</td>
<td>1.2905</td>
<td>1.2905</td>
<td>-0.0005</td>
<td>-0.41%</td>
<td>2,210,550</td>
</tr>
<tr>
<td>U.S. DOLLAR BILL 3 YR</td>
<td>1.2950</td>
<td>1.2960</td>
<td>1.2950</td>
<td>1.2955</td>
<td>1.2955</td>
<td>1.2955</td>
<td>1.2955</td>
<td>-0.0005</td>
<td>-0.38%</td>
<td>1,810,850</td>
</tr>
<tr>
<td>U.S. DOLLAR BILL 5 YR</td>
<td>1.3000</td>
<td>1.3010</td>
<td>1.3000</td>
<td>1.3005</td>
<td>1.3005</td>
<td>1.3005</td>
<td>1.3005</td>
<td>-0.0005</td>
<td>-0.39%</td>
<td>1,508,250</td>
</tr>
<tr>
<td>U.S. DOLLAR BILL 7-1/2 YR</td>
<td>1.3050</td>
<td>1.3060</td>
<td>1.3050</td>
<td>1.3055</td>
<td>1.3055</td>
<td>1.3055</td>
<td>1.3055</td>
<td>-0.0005</td>
<td>-0.38%</td>
<td>1,200,850</td>
</tr>
<tr>
<td>U.S. DOLLAR BILL 10 YR</td>
<td>1.3100</td>
<td>1.3110</td>
<td>1.3100</td>
<td>1.3105</td>
<td>1.3105</td>
<td>1.3105</td>
<td>1.3105</td>
<td>-0.0005</td>
<td>-0.38%</td>
<td>900,050</td>
</tr>
<tr>
<td>U.S. DOLLAR BILL 15 YR</td>
<td>1.3150</td>
<td>1.3160</td>
<td>1.3150</td>
<td>1.3155</td>
<td>1.3155</td>
<td>1.3155</td>
<td>1.3155</td>
<td>-0.0005</td>
<td>-0.38%</td>
<td>600,050</td>
</tr>
<tr>
<td>U.S. DOLLAR BILL 20 YR</td>
<td>1.3200</td>
<td>1.3210</td>
<td>1.3200</td>
<td>1.3205</td>
<td>1.3205</td>
<td>1.3205</td>
<td>1.3205</td>
<td>-0.0005</td>
<td>-0.38%</td>
<td>300,050</td>
</tr>
</tbody>
</table>

*Contracts deliverable months that are currently traded*

Prices represent the open, high, low, and settlement (or closing) price for the previous day.

One day's change in the settlement price:

- **Change (Open - Close)**:
  - The number of contracts still in effect at the end of the previous day's trading session. Each contract represents a buyer and a seller who will have a Contract position.
  - One day's change in the futures interest rate—equal and opposite to change in the settlement price.
  - The settlement rate implied by the settlement price, e.g., 100 - 96.18 = 3.82.
  - The total of the right column, and the change from the prior trading day.

From Equation (7-2) it follows that some movements in the base particular asset are predictable movements, based on the cost of the asset.

The first of these predictable movements is the convergence futures price to the price implied by the cost-of-carry relation. We keep in mind that the cost-of-carry model is an equilibrium model; the futures price only stays at the price implied by Equation (7) if arbitrage forces will act to bring the futures price back to that price by the cost-of-carry model. Thus, over the tenor of the futures contract, futures prices will tend to converge toward the price implied by the cost-of-carry relation.

The second predictable movement is the convergence of the futures price to the leisure price at expiration of the futures contract. As delivery becomes shorter, the cost of carry declines. Storage maintenance costs decrease along with the time for storing the collateral. The shorter the holding period, the lower the opportunity cost of holding the asset. Hence, as specified by Equation (7-2), as the delivery becomes shorter and the cost of carry, \( c \), becomes smaller, futures price, \( F \), converges to the cash price, \( P \).

**Futures Prices and Expected Future Spot Prices**

We have seen that the price today for a futures contract's delivery at period \( T \) is related to the prevailing spot price via the cost-of-carry. However, a more important question is how the price of the futures contract specifying delivery at period \( T \) is related to the spot price at period \( T \).

The expectations model states that the current futures price to the market's expected value of the spot price at period \( T ")

\[
F_T = E(P_T)
\]

If this model is correct, a speculator can expect neither to lose from a position in the futures market; expected profits are

\[
E(\text{profit}) = E(P_T) - F_T = 0
\]

Put another way, if the expectations model is correct, the speculator can expect to earn only the riskless rate of return. This counterintuitive idea can best be understood through an example.
Example

The expectations model

Suppose that, at time period $t$, a speculator purchases a futures contract at a price of $F_0$ and posts 100% margin in the form of riskless securities. At contract maturity, at time $T$, the value of the margin account will have grown to $F_0(1 + r_f)$.

where $r_f$ is the risk-free rate of return for a period equal to the maturity of the futures contract.

At maturity, the value of the futures contract itself will be $F_T = F_0(1 + r_f)$.

The actual rate of return the speculator will earn is

$$r = \frac{(1 + r_f)F_0 - (F_T - F_0)}{F_0} = 1 + r_f + \frac{(F_T - F_0)}{F_0}$$

The expected rate of return the speculator will earn is

$$E(r) = r_f + \frac{E(F_T) - F_0}{F_0}$$

Hence, if the expectations model is correct, the expected rate of return is

$$E(F_T) = F_0 \cdot E(r) = r_f$$

Proponents of the expectations model argue that, in a market with rational traders, the expectations model simply has to work. The argument behind this position goes something like this:

If a majority of the traders expect the spot price at maturity to be above the prevailing futures price, they would buy futures, thereby forcing the futures price up. Conversely, if these rational traders expected the spot price in the future to be below the current futures price, they would sell futures, lowering the futures price. Hence, the only price that will give an equilibrium is for the futures price to be equal to the expected spot price at maturity.

Cost of Carry versus Expectations

We have presented conflicting views of the way in which futures prices are formed. Suppose the expected future price does not change over the maturity of the futures contract; price expectations remain constant. The
expectations model would yield a constant futures price, as illustrated by the horizontal line in Figure 7-4.

John Maynard Keynes was among the first to take exception to such a model. He looked at commodity futures and argued that the futures contract provides a mechanism to transfer risk from the hedgers (the commodity producers, who have natural long positions in the commodity) to speculators. To accomplish this transfer of risk, the equilibrium in the futures market would be such that hedgers would be (on net) short commodity futures contracts (to offset their natural long positions), while speculators would be long commodity futures contracts. Consequently, to get the speculators to buy the commodity futures contracts—to hold the long positions in futures—the expected rate of return for holding futures would have to exceed the risk-free rate. For the expected rate of return on the futures position to exceed the risk-free rate, the futures price would have to be less than the expected spot price and rise as the


Figure 7-4. Futures Prices over Time with Constant Future Price Expectations.

<table>
<thead>
<tr>
<th>Normal contango</th>
<th>Normal backwardation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contract origination</td>
<td>Time</td>
</tr>
<tr>
<td>T</td>
<td>Contract maturity</td>
</tr>
</tbody>
</table>
contract maturity approaches. This relation, referred to by Keynes as normal backwardation, is illustrated as the rising line in Figure 7.4.

Conversely, if equilibrium is achieved through hedgers, on net, being long futures contracts, the speculators would have to be enticed to hold short futures positions—in net futures contracts. In this case the futures price would have to begin above the expected spot price and fall as contract maturity approaches. Referred to by Keynes as normal contango, this relation is illustrated by the falling line in Figure 7.4.

Example

Backwardation and contango in bond futures

Consider again the construction of a synthetic futures position, this time a futures position on bonds. A party borrows (at the short-term rate) and uses proceeds to purchase bonds (yielding the long-term rate).

The cost of this synthetic bond futures contract is the rate that must be paid on the short-term borrowing minus the rate earned on the coupon received on the long-term bonds. Through arbitrage, the futures price in a standard contract must be equal to the cost of the synthetic futures.

If the yield curve is positively sloped, the cost of this synthetic futures is negative; the long-term yield received exceeds the short-term yield paid. Hence, the expected return to the holder of the long futures position is positive. This is normal backwardation; the futures price is less than the expected future spot price, and futures prices are known for contracts whose delivery dates are further in the future. Look again at Figure 7.3—specifically, at the treasury bond futures traded on the Chicago Board of Trade. The further in the future is the delivery date, the lower is the futures price.

<table>
<thead>
<tr>
<th>Month</th>
<th>85.06</th>
</tr>
</thead>
<tbody>
<tr>
<td>March</td>
<td>95.05</td>
</tr>
<tr>
<td>June</td>
<td>92.20</td>
</tr>
<tr>
<td>Sept</td>
<td>91.16</td>
</tr>
<tr>
<td>Dec.</td>
<td>90.24</td>
</tr>
<tr>
<td>March 89</td>
<td>89.01</td>
</tr>
<tr>
<td>June 89</td>
<td>89.12</td>
</tr>
<tr>
<td>Sept. 89</td>
<td>81.25</td>
</tr>
</tbody>
</table>

If the yield curve is inverted—negatively sloped—the reverse holds true. The cost of the synthetic futures is positive, so bond futures prices would have exhibit normal contango.

The cost of the synthetic futures is positive, so bond futures prices would have exhibit normal contango.
Stephen Figlewski argues that it is the cost-of-carry model, rather than the expectations model, that explains the manner in which futures prices are determined. He argues that the expectations model fails to take into account arbitrage. His argument can be paraphrased as follows:

If an individual is holding treasury bonds that could be delivered against a treasury bond futures contract, and if the futures price is equal to this individual's expectation of the future spot price of treasury bonds, an arbitrage is possible. He could sell futures against his cash position, thereby eliminating all of the risk in his position, but the expected return would be the same as it would have been held the risky bond position unhedged. This means that the expected return he is earning exceeds the risk-free rate. He is earning a risk premium without bearing the risk. Such a situation would attract other investors, and as more people tried to buy bonds and sell futures, the cash price would be bid up and the futures price bid down until the cost-of-carry relation is reestablished.

However, this should not be taken to mean that the cost-of-carry relation ignores expectations; it does not. The market's expectation of the future spot price of the asset is implicit in the current spot price. In an efficient market—and everything we have seen suggests that these financial markets are efficient—the price today incorporates all available information, including information about what the asset will be worth in the future. Following Figlewski, "the point of the cost of carry model is that given the current cash price [which incorporates a forecast for future spot prices], expectations about the cash price at expiration should not have any independent effect on futures prices."

Futures Prices and the Cost of Hedging: Basis

Given what we have said about hedging in general, it follows that, as long as futures are priced according to the cost-of-carry relation, the total return to the holder of a fully hedged position should be the risk-free rate of return. The cash position is the underlying asset is a risky position, so the market return for holding this position would be made up of the risk-free rate of return plus a risk premium. By selling a futures contract against the underlying exposure, the hedger has transferred the riskiness of the asset to the buyer of the contract, so the buyer should

7. Ibid. p. 69.
turn the risk premium. Hence, the person who holds the asset and has hedged completely by selling a futures contract against the asset is left with the riskless rate of return.

The problem is that the underlying cash position may not be fully hedged; the return to the futures contract may not be exactly equal to the risk premium on the underlying asset. The hedger has a long position in the asset and a short position in the futures contract and has consequently invested in the difference between these two assets. Hence, the return to the hedged portfolio is determined by what happens to the difference between the spot and the futures prices—which is what we defined earlier as the basis:

$$\text{Basis} = F - P$$

The hedger, the person with the long position in the asset and the short position in the futures contract, profits if the basis gets smaller and loses if the basis gets larger. The reverse is true for the speculator.

Hence, the hedger has not eliminated all risk but has instead replaced price risk with basis risk. The reason that hedgers use the futures market is that basis risk is potentially more manageable than price risk. But managing the basis risk requires understanding the sources of this risk.

Basis risk results from unpredictable movements in the basis—unpredictable differences between the futures price and the spot price. Although these unpredictable movements in the basis arise from various sources, there are four primary sources of basis risk: changes in the convergence of the futures price to the cash price, changes in factors that affect the cost of carry, mismatches between the exposure being hedged and the futures contract being used as the hedge, and random deviations from the cost-of-carry relation. 

The Convergence of the Futures Price to the Cash Price. A "normal" pattern of convergence of the futures price to the cash price is illustrated in Figure 7.2: Panel (a) illustrates the spot and futures prices.

---

1. Predictable movements in basis—such as the convergence of the basis toward the cost of carry during the term of the futures contract and the convergence of the basis to zero at contract maturity, which were discussed earlier in this chapter—are incorporated in the expected return at the hedged position.

2. Another way of looking at this is to say that the basis for financial instruments is affected by six factors: the cost of carry, the time until delivery, the deliverable supply, the cost of delivery, changes in cash instruments (e.g., coupons and maturity), and price expectations.
themselves, and Part (b) illustrates the basis—the difference between these two prices. At contract maturity, the futures price and the cash price merge; thus, for a hedge in which the futures contract is held to maturity, the return on the futures position will be equal to the return on the asset itself. However, if the futures position is unwound prior to contract maturity, the return from the futures position could differ from the return on the asset due to the basis risk. And, as is obvious from Figure 7.3, the basis is in large part determined by the rate and path of convergence of the futures price to the spot price. Moreover, the convergence determines the behavior of the margin account. For the case illustrated, the futures price rises smoothly over time. Thus, there would be a gradual flow of margin from the account of the party who sold the futures contract to the account of the buyer of the futures contract.

Suppose that the path of convergence was different from that in Figure 7.5. If the futures price converged more rapidly than is illustrated, the basis would decline toward zero more quickly and would consequently be smaller at any point in time. In this case, the flow of margin from the futures seller to the futures buyer would occur more rapidly.10

Regardless of the speed of convergence, the total amount of margin that moves from one seller of the contract to the buyer of the contract is the same; the only difference is in the timing of the cash flows. The slower the futures price converges to the spot price, the slower are the transfers of the margin. Consider an extreme situation, where the spot price moves as before, but the futures price does not move at all until just before maturity:

With the convergence as illustrated, no monies are transferred from the contract seller to the contract buyer until just before contract maturity. In such a case the futures contract would behave essentially like a forward contract.
Figure 7.5: Convergence of the Future Price.
Alternatively, suppose, as illustrated in Figure 7-6, that the spot price overshoots its equilibrium. In this situation, the basis is negative for a time. Margin would first move from the seller to the buyer, then from the buyer to the seller, again from the seller to the buyer.\footnote{However, since the starting and ending points for the futures price are the same.}
Changes in Factors Affecting the Cost of Carry. Clearly, as the cost of carry changes, the basis on a futures position changes. For commodity futures, the cost of carry includes storage and insurance costs; changes in either of these cause the basis to change. However, the most significant determinant of the cost of carry is the interest rate. As the interest rate increases, the opportunity cost of holding the asset rises, so the cost of carry—and therefore the basis—rises.

Mismatches between the Exposure Being Hedged and the Futures Contract Being Used as the Hedge. So far, we have implicitly been examining situations in which the exposure being hedged is the same as the futures contract, for example, hedging an exposure to a movement in the deutsche mark dollar exchange rate with a deutsche mark futures contract, or hedging an exposure to the U.S. treasury bill interest rate with treasury bill futures. However, situations exist in which the position being hedged does not match the deliverable asset/commodity for any futures contract, and the hedger will have to rely on a cross-hedge. For example, a deutsche mark futures contract might be used as a cross-hedge for an exposure in Swedish krona, or a treasury bill futures contract might be used as a cross-hedge against an exposure to the U.S. commercial paper rate.

In a cross-hedging situation, there is an additional source of basis risk. Basis results not only from differences between the futures price and the prevailing spot price of the deliverable asset, but also from differences between the spot prices of the deliverable asset and the exposure being hedged:

\[ \text{Basis}_{\text{cross-hedge}} = (F_y - P_y) + (P_y - P_t) \]

For example, for an exposure to the U.S. commercial paper rate hedged with a treasury bill futures, basis could result from (1) differences in the futures and spot prices of treasury bills, and (2) differences in the spot treasury bill and commercial paper interest rates. Let's define the latter contribution to basis risk as the cross-hedge basis. As will be shown in Chapter 8, the "normal" cross-hedge basis can be determined by the correlation between the spot price of the asset being hedged and the spot price of the deliverable product in the futures contract being used as the

---

12 We will spend a considerable amount of time talking about the way a cross-hedge is constructed and managed in the next chapter.
hedge. Stephen Figlewski\textsuperscript{13} has argued that there are three important factors responsible for variation in the basis for a cross-hedge:

1. **Maturity mismatch**: The maturity of the underlying instrument for the futures contract may be different from the maturity of the asset underlying the exposure. For example, the exposure of the S&L we have alluded to may be to thirty-year mortgage interest rates, and the hedge may be constructed using treasury bond futures, for which the underlying instrument has a maturity of fifteen years.\textsuperscript{14}

2. **Liquidity differences**: Suppose the asset being hedged is traded in a market that is illiquid in comparison to the market for the deliverable asset in the futures contract. In such an instance, the price fluctuations for the asset being hedged would be likely to be large relative to the fluctuations in the price of the deliverable asset, implying that basis would increase. Hence, the basis is inversely related to the liquidity for the asset being hedged.

3. **Credit risk differences**: An example provided by Figlewski is the use of treasury bond futures to hedge a portfolio of corporate bonds. For another way, changes in the quality spread show up in the basis for a cross-hedge.

**Random Deviations from the Cost-of-Carry Relation.** The final source of basis risk is the catchall component. From day to day and from minute to minute, the basis on a futures position will change for reasons not understood; however, over longer periods, this random, “white noise” component of basis risk will cancel itself out.

\textsuperscript{13} Figlewski, op. cit. pp. 77-78.

\textsuperscript{14} As noted in the preceding chapter, the bond to be delivered against a T-bond futures must have at least fifteen years remaining to maturity.