The Forward Contract

As indicated in Chapter 3, the forward contract is the fundamental building block of the financial instruments. It forms the basis for the risk management instruments both from a conceptual standpoint and with respect to their introduction and use in the financial markets. Most notable are foreign exchange forward contracts, the contracts with which banks, corporations, governments, and some individuals manage their exposure in the $1.5 trillion foreign exchange markets (estimated daily volume during peak periods).

Simply put, a forward contract is a contract made today for the delivery of an asset in the future. The buyer of the forward contract agrees to pay a specified amount at a specified date in the future in order to receive a specified amount of a currency, a specified amount of a commodity, or a specified coupon payment from the counterparty. The specified future price is the exercise price of the contract.1

To make this definition of a forward contract more concrete, we have provided some illustrations in Figure 4-1. Part (a) illustrates a foreign exchange forward. The party illustrated has agreed to pay, at time $T$, $Y$ in order to receive $X$. Part (b) illustrates a commodity forward contract. The party illustrated has agreed to pay, at time $T$, $Y$ in order to receive $X$ barrels of oil. Part (c) illustrates an interest rate forward contract. The party illustrated has agreed to pay a fixed-rate coupon, $R$, at time $T$, to receive a floating-rate coupon, $R$.

At contract origination, the present value of an at-market forward contract is zero. The exercise price of the contract is set at the expected future price, so neither the buyer nor the seller of the forward will

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1. The forward contract may stipulate that only the difference between the exercise price and the spot price prevailing at the future date be exchanged.
Figure 4.1. Some Illustrative Forward Contracts. (a) A Foreign Exchange Forward (b) A Commodity Forward. (c) An Interest Rate Forward.
obtain value unless the exchange rates, commodity prices, or interest rates differ from expectations. Thus, the exercise price set is the price that equates the buyer's expected cash flow to the seller's expected cash flow.

Since the net present value of the contract at origination is zero, why do parties enter into forward contracts? The contract must certainly have value for both the buyer and the seller; otherwise they would not enter into the contract. The value exists because one party desires to reduce risk or has a view on the future movement of the underlying price series that differs from the market's; or the value may exist because the forward contract allows creation of something valuable (e.g., synthetic assets).

In the main, participants in the forward markets are those who wish to fix future transaction costs. For example, a car importer knows today's cost of a particular model of car in both the domestic and the foreign currency. The importer also knows what the car purchased today will sell for upon delivery in domestic currency. What is uncertain is how much the car will cost in domestic currency when it arrives, since it must be paid for in a foreign currency at that time. To eliminate this uncertainty, the importer today can contract the price of exchange between domestic currency and foreign currency for this future date using a forward contract.

The automobile importer also provides a more down-to-earth example. When an importer sells a car, it is unusual that cash is paid and the keys are handed over simultaneously. In most auto sales, the purchaser agrees to pay and the seller agrees to deliver at some date in the future (the completion date of the sale) at the agreed-upon price. That price will be paid on the completion date but is agreed upon now, and it will not change regardless of what happens to auto prices in the interim. The contract covering the period from agreement to completion is, in fact, nothing more than a forward contract. If the time to completion of the sale is one week, no one really gives the forward nature of this contract much consideration, but if the time to completion is several months, car prices could change significantly. In such a circumstance, the forward contract is a valuable way of reducing price risk for both parties. Forwards, in this sense, are a part of everyday life.

The number of people interested in using forward contracts is therefore determined by the number who face uncertainty about future prices. Since a great number of people are exposed to this kind of uncertainty, why isn't there a forward contracting agent on every street corner con-
tracting in every commodity and every consumer good. As we will see, the answer lies in the potential for default.

**Forward Contracts and Default**

In a world of no transaction costs, with all parties always paying their debts, there might indeed be a forward contracting agent on each street corner. Unfortunately, not all parties to contracts fulfill their responsibilities without coercion. Moreover, the act of contracting itself entails a cost. Thus, the forward contract exhibits both performance risk and transaction costs. Since transaction costs for a forward contract are usually small, we concentrate our attention on performance risk—or, in bankers' terms, "credit risk."

Forward contracts are, by definition, credit instruments. Suppose today you contract to deliver the difference between (1) a specified ratio of dollars per deutsche mark and (2) the market ratio of dollars per deutsche mark one year in the future, multiplied by some specified number of dollars. Say the exercise price of exchange is 3.00 DM/$ (the dollar price of a DM is $0.33). If at contract maturitiy the exchange rate has risen to 4.00 DM/$ (the price of a DM has fallen to $0.25), you are now richer. You can buy deutsche marks for $0.25 and then sell them for $0.33—a profit of $0.08 per DM. Your counterparty—the party who agreed to buy the DM—is now poorer by the same amount. If the buyer decides to abrogate the contract, you are out a sum of money, just as would be the case if a loan were renegotiated. In this sense, a forward contract is a credit instrument.

The fact that forward contracts entail credit risk is important in determining who uses them. Individuals, institutions, corporations, and governments with access to credit lines are able to use forward contracts. Those for whom the costs of creating credit lines are high relative to the benefits of using the forward contract will not participate in the market. Realistically, then, the forward market is not appropriate for the individual, the sole proprietorship, or the small corporation. It is a market for

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2. Transaction costs are small as a percentage of the forward contract amounts normally seen in the market. Transactions have a high fixed-cost component and very low marginal costs. Thus, a $100 and a $10,000,000 forward foreign exchange contract have nearly the same transaction cost. Logically, for a $10,000,000 transaction, the transaction cost component is of much less importance than for a $100 transaction.
large corporations, governments, and other institutions—both financial and nonfinancial—that have access to credit lines as a regular part of their business.

A Foreign Exchange Forward Contract

The Contract

Foreign exchange spot markets and foreign exchange forward markets are liquid, efficient, and sophisticated. The movement of these markets is therefore largely regulated by the legal contract under which they operate and by the enforceability of that contract.

As was just noted, forward contracts entail performance risk. Therefore, the forward contract is written to address this risk. In this sense, the contract is similar to that of a loan or a line of credit. For example, the contract defines payment responsibility following contract expiration. Continuing our example of the previous section, suppose the contract stipulates that the price of future delivery is at a rate of DM 3.00 per dollar. If the price of dollars is DM 2.00 at contract maturity, the party selling deutsche marks for dollars at DM 3.00 would have an incentive to break the contract by not delivering. Consequently, the forward contract is written in such a way that nonperformance amounts to not making a payment on a loan.

Agreeing on an exercise price is left to the two parties, but a forward contract is clear about how the price is referenced. In the case of foreign exchange forward contracts, the contract reference is a set amount of one currency for a set amount of another—a forward rate of exchange. The contract specifies the spot rate at the maturity date as the average of the bid and asked prices quoted by a specified bank for the spot purchase and spot sale, respectively, of the contract currency in exchange for U.S. dollars at a prescribed location (usually New York, London, or Tokyo) at a prescribed time (usually 11 A.M. local time).

The process of setting the exercise price and settlement date of the contract (with written confirmation) is an essential part of forward contracting. The settlement date of a contract is the date at which a

3. We would like to thank our colleagues at the Chase Manhattan Bank, particularly Sandra Zusmaner and Richard Zeich, for providing us with specific information on the terms of a forward contract.
contract is actually payable. For example, if on March 1 a three-month forward is agreed on, the maturity date would be June 3, but the settlement date would be two days later. These dates, as well as the date of origination of the contract, are stipulated in a confirmation telex exchanged between the contracting parties.

Most foreign exchange forward contracts specify that neither party is obligated to actually deliver one currency against another. Rather, the amount payable is determined by the difference between the spot price and the exercise price at contract maturity. The payoff profile for forward contract positions is illustrated in Figure 4-2. Part (a) shows the payoff profile for the buyer of the forward contract—the individual who is long the forward contract. At contract maturity, the buyer is obligated to buy the asset at the price agreed to at contract origination, $P^f_T$. If the spot price at maturity, $P_T$, exceeds the exercise price, the forward contract owner will be able to buy the asset at the lower exercise price and sell at the higher spot price, making a profit of $P_T - P^f_T$ per unit. Hence, the profit for the owner of the forward contract is

$$\text{Profit} = (P_T - P^f_T) \times \text{(Number of units contracted)} \quad (4-1)$$

Conversely, the seller of the forward contract profits when the spot price at maturity is less than the exercise price: the seller can buy at the cheaper spot price and sell at the higher exercise price. The profit for the seller of the forward contract is

$$\text{Profit} = (P^f_T - P_T) \times \text{(Number of units contracted)} \quad (4-2)$$

Because the foreign exchange forward contract is settled on the difference between the spot exchange rate at $T$ and the contracted exchange rate, the foreign currency itself does not have to be delivered; the contract can be settled in U.S. dollars. Exactly as in any loan agreement, if one party is late in delivery of funds, a penalty interest cost is incurred on the outstanding balance.

Not surprisingly, the clauses of a contract that concern pricing and settlement comprise the minority of the forward contract; the bulk of the documentation is involved with the credit issues just mentioned. Events

4. The reason for this is that spot currency transactions are dealt two days forward (with the exception of Canadian dollars for U.S. dollars), and forwards are always quoted from spot. Thus, a March 1 spot quote is for March 3, so a three-month forward would be March 3 to June 3.
Figure 4-2. The Payoff for Forward Contract Positions. (a) The Profile for the Buyer (the Long Position).
(b) The Profile for the Seller (the Short Position).

At contract maturity (time = T), the profit to the buyer of a forward contract is equal to the difference between the spot price at T and the exercise price agreed to at contract origination ($F_T - F_0^f$) times the size of the forward contract. The profit to the seller of the contract is the reverse.
of default receive particular attention, underscoring the credit nature of the contract.

The essence of a forward contract is the fact that the maturity date (the date on which the contract expires) is the only date that is relevant in calculating the amount one party will owe the other upon settlement. That is, the legal agreement stipulates that the settlement flows are based on the deviation in contract price from the spot price on the maturity date. From the contract's perspective (but not necessarily from the contracts' perspectives) the time path that the foreign exchange rate follows between the origination date and the maturity date (when the settlement payment is calculated) is of no consequence.

The Forward Foreign Exchange Rate

We turn next to the pricing of a foreign exchange forward contract—the determination of the forward foreign exchange rate. This section is intended not only to provide an understanding of the pricing of foreign exchange forwards but also to illustrate the basic concepts of a forward contract. At origination, an at-the-money forward contract has a net present value equal to zero. Given this condition, alternative market constraints can be used to create arbitrage pricing models for a forward contract.

 forwards are traded in most major currencies, with bid-asked spreads quoted in standard maturities of one, two, three, six, nine, and twelve months. Moreover, for the major currencies—sterling, yen, and deutsche mark—quotes on four, five, etc. months are also available. On a negotiated basis, forwards are also available in major currencies for odd dates (also referred to as broken dates). The extent to which a currency forward is available depends on exchange controls, the depth of alternative markets, and the monetary policy of a country. Because of regulatory differences among domestic markets, the reference market used to price a forward (set a forward rate for a currency) is usually the Euromarket.

The easiest way to price a foreign exchange rate forward contract is to determine the future foreign exchange rate that could be created synthetically. The technique used to do this is called covered arbitrage. (Alternatively, one could think of this technique as the methodology of

5. Longer-term contracts do exist, in which case the bid-asked spreads are subject to negotiation.
creating a forward rate that guarantees that the contract's net present value is zero.) For clarification, think of a forward contract as a pair of zero-coupon loans.

This concept is illustrated in Figure 4.3. Part (a) illustrates the cash flows of a party who has agreed to buy deutsche marks forward (or, conversely, sell dollars forward). This contract obligates the party to pay, at period T, a set number of dollars in return for a set number of deutsche marks. Part (b) compares the cash flows in part (a) to a pair of zero-coupon loans. At time zero (i.e., at contract origination) the party illustrated borrows in dollars, $Q$, and simultaneously lends the same amount in deutsche marks, DM R. That is, given the spot exchange

![Figure 4.3. A Forward Contract as a Pair of Loans.](image-url)
rate at time zero between dollars and deutsche marks. $S_0$, $Q$ dollars is equivalent to $P$ deutsche marks: $DM \times P = SQ \times S_0$. At maturity (time $T$) the party will have to pay back $Q \times (1 + r_Q)$ dollars, where $r_Q$ is the U.S. dollar interest rate for maturity $T$, and will receive $R \times (1 + r_{DM})$ deutsche marks, where $r_{DM}$ is the $T$-period rate for a deutsche mark borrowing. If the two cash flow diagrams in part (b) were combined into a single cash flow diagram, the result would be the forward contract illustrated in part (a), where

$$DM \times X = R \times (1 + r_{DM}) \quad \text{and} \quad SY = Q \times (1 + r_Q)$$

Consequently, a forward contract for foreign exchange can be priced as if it were a pair of zero-coupon loans; the bullet repayments plus interest are netted against the future spot foreign exchange rate on the maturity date. Thus, on a purely mechanical pricing basis, the spot rate at origination ($S_0$) times the amount of the domestic currency ($SQ$) fixes the amounts of foreign currency to be lent. And with knowledge of the two relevant interest rates, $r_Q$ and $r_{DM}$, the amounts of the two cash flows to be exchanged at $T$ are determined.

Therefore, the forward exchange rate at contract origination, $F_0$, can be obtained by dividing one cash flow at $T$ by the other:

$$F_0 = \frac{S \times (1 + r_{DM})}{Q \times (1 + r_Q)} \quad (4-3)$$

Since $P$ deutsche marks is equal to $Q$ dollar times the spot exchange rate at contract origination ($R = Q \times S_0$), it follows that the forward rate is:

$$F_0 = S_0 \left( \frac{1 + r_{DM}}{1 + r_Q} \right) \quad (4-4)$$

Generalizing, we can express the forward exchange rate for currencies $i$ and $2$ at time $t$ as:

$$F_t = S_t \left( \frac{1 + r_{2,t}}{1 + r_{i,t}} \right) \quad (4-5)$$

where the forward and the spot rates are defined at time $t$ as the number of currency $2$ units per country $1$’s currency unit (i.e., if country $2$ is West Germany, $F_t$, and $S_t$ are defined in deutsche marks per dollar).

Alternatively, Equation (4-5) can be written in the form often referred to as interest rate parity:
\[ \frac{F_t}{S_t} = \frac{1 + r_2}{1 + r_1} \]

That is, the ratio of the forward rate to the spot is a reflection of the interest rates in the two countries. If the interest rate in country 2 is higher than that in country 1, then the forward rate is greater than the spot—or country 2’s currency is weaker in the forward market than country 1’s. Returning to dollars and deutsche marks, if the dollar interest rate is lower than the DM interest rate, the forward DM/$ rate is greater than the spot; an alternative way of saying this is that the dollar is selling at a premium to the deutsche mark (or the DM is selling at a discount to today’s spot rate).

**Example**

**Calculating a forward exchange rate**

Foreign exchange and interest rates as of March 15, 1988, were as follows:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot DM per dollar</td>
<td>1.6685</td>
<td></td>
</tr>
<tr>
<td>U.S. LIBOR (1 year)</td>
<td>0.0730</td>
<td></td>
</tr>
<tr>
<td>DM LIBOR (1 year)</td>
<td>0.0357</td>
<td></td>
</tr>
</tbody>
</table>

Using Equation (4.5), we can calculate the one-year forward rate as 1.6105. The spot rate minus the forward rate is called the premium if positive and the discount if negative. If the forward is less than the spot, the DM is anticipated to be stronger in the future, if the forward is greater than the spot, the DM is anticipated to be weaker in the future. In this example, the premium is 0.0580. Consequently, as of March 15, 1988, the DM is expected to be stronger against the dollar in one year than at present.\(^6\)

**Bid-Ask Spreads and Forwards**

In the preceding calculations of the forward rate from arbitrage conditions between markets, we used what are commonly referred to as *mid-rates*. The problem is that buyers must pay an amount such that the sellers will earn enough to cover the transaction costs of creating the synthetic forward contract. After all, who would want to spend their

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6. The use of the word *expected* might imply to some readers a theory of exchange rate determination. We do not intend it to be utilized thus. Rather, we wish to convey that the forward rate is contracted in such a way, given existing market forces, to imply a stronger future DM spot price.
time providing a market for forwards unless they were paid to do so? A market exists to bring different parties together to trade, but the provider of the market must be compensated.

The way the foreign exchange market compensates the banks providing this service is through the bid-ask spread. A market participant specializing in making prices in a particular contract—a market maker—will quote one price to a person if that person buys and another price if that person sells. This bid-ask spread compensates for the cost involved in providing the service of contracting.

In the foreign exchange market, prices are usually quoted in units of foreign currency per dollar. Prices are quoted to the fourth decimal point for deutsche marks, sterling, Canadian dollars, etc. and to the first or second decimal place for currencies such as Italian lira (trading at 1.275.6 per dollar on March 16, 1987) and yen. The bid-ask spread on the major currencies in the spot market is very small. For example, on spot deutsche marks, the bid-ask spread is usually as low as 5 to 10 "pips" (a pip is 1/10,000 of a DM). Thus, on March 15, 1988, at 11:12 GMT Chase Manhattan Bank was showing a quote of "DM/$ 1.6680/90" or "DM/$ 1.6680-1.6690."

The interest rate markets (the money markets) also operate on this bid-ask concept. For example, on March 16 at 16:11 GMT the twelve-month Eurodollar deposit rate was 7 1/2% to 7 1/4%, and the Euro deutsche mark deposit rate was 3 1/2 to 3 3/4%. Given this information, we can look again at the forward rate and the forward premium described in the last example; but this time, instead of using mid-rates, we calculate the relevant bid-ask forward rates.

**Example**

**Recalculating a forward exchange rate**

On March 15, 1988 the relevant bid-ask spreads were:

<table>
<thead>
<tr>
<th>Currency</th>
<th>Bid</th>
<th>Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot DM per dollars</td>
<td>1.6680</td>
<td>1.6690</td>
</tr>
<tr>
<td>U.S. (1 year)</td>
<td>0.071475</td>
<td>0.073125</td>
</tr>
<tr>
<td>DM Libor</td>
<td>0.010000</td>
<td>0.010625</td>
</tr>
</tbody>
</table>

7. The exceptions to this rate are the pound sterling and the ECU (European Currency Unit), which are usually quoted in dollars per unit of sterling or ECU.
8. In this case we refer to the Euro-money markets, for example, the Eurodollar and Euro deutsche mark markets.
Utilizing Equation (4-5), we calculate the forward rate as $1.6087-1.6135$. To obtain the twelve-month bid rate for the forward, we divided $(1 + r_{DM} \text{ bid})$ by $(1 + r_D \text{ ask})$ and then multiplied by the spot bid. The reverse was done to obtain the forward ask rate. Let’s walk through the logic. Since we are attempting to create a future buying rate, we want to pay a rate less than the rate at which we would sell. Thus, we take the buying rate today as our base—the bid spot rate. We must then invest deutsche marks for one year while borrowing dollars. Since we are investing deutsche marks, we can only get the buying rate—the bid rate—and since we are borrowing dollars, we get stuck with having to pay the higher rate. To calculate the ask rate, we reverse the process.

In this example we calculate the premium as 0.0593-0.0555. Note that the premium is quoted with the higher number first. Usually the foreign exchange forward market quotes the spot and then the forward premium or discount. If the currency is at a premium to the dollar, as in this case (DM interest rates are lower), then the higher figure is quoted first; this will indicate that the numbers are subtracted from the spot bid-ask spread.

To give some idea of how these prices actually appear in the market, Figure 4-4 provides the spot and forward rates for Chase Manhattan Bank foreign exchange trading operations from around Europe as they appeared on Reuters on March 16, 1985.

**Forward Contracts on Interest Rates**

As noted in Chapter 1, on October 6, 1979, the Federal Reserve shocked the financial system by raising the discount rate 200 basis points and starting to target the money supply. From this date, interest rate volatility became the norm, providing the impetus for forward contracts on interest rates, referred to as forward rate agreements (FRAs). The market

9. Note in Figure 4-4 that the quote of the forward premium for deutsche marks differs slightly from the calculations in the preceding example. The reason is that the interest rates used in the example were obtained one hour earlier than this table was obtained. This illustrates that to actually do an arbitrage, one must be able to move quickly, before someone else in the market does it. There is another reason why arbitrage situations don’t exist all the time: the bid-ask spread in the forward market will likely be narrower than the arbitrage range spread. This is because the credit risk inherent in the forward contract is less than that implied in the actual borrowing and deposit of funds that would be necessary to perform the arbitrage.
Figure 4-4. Prices in the Foreign Exchange Market, Quoted on March 16, 1986 (Taken from Reuters Screen CMB3).

<table>
<thead>
<tr>
<th>Location</th>
<th>Spot</th>
<th>1Mo</th>
<th>2Mo</th>
<th>3Mo</th>
<th>6Mo</th>
<th>12Mo</th>
</tr>
</thead>
<tbody>
<tr>
<td>LDN STG</td>
<td>1.8800/10</td>
<td>32.30</td>
<td>62.80</td>
<td>95.92</td>
<td>132/176</td>
<td>328.318</td>
</tr>
<tr>
<td>FFM DMK</td>
<td>1.6680/90</td>
<td>49.44</td>
<td>97.92</td>
<td>147.142</td>
<td>250/280</td>
<td>500.575</td>
</tr>
<tr>
<td>FFM SFR</td>
<td>1.3900/10</td>
<td>51.50</td>
<td>117.124</td>
<td>174/171</td>
<td>327/321</td>
<td>613.305</td>
</tr>
<tr>
<td>PAR FFR</td>
<td>5.6000/00</td>
<td>62.72</td>
<td>130/140</td>
<td>120/245</td>
<td>426/440</td>
<td>715.776</td>
</tr>
<tr>
<td>BUC HIC</td>
<td>1.75/72</td>
<td>4.70</td>
<td>7.9</td>
<td>15/15</td>
<td>36/30</td>
<td></td>
</tr>
<tr>
<td>MLT LIT</td>
<td>1.2377,75</td>
<td>470/720</td>
<td>9900</td>
<td>1335/14</td>
<td>25/26</td>
<td>4420/47</td>
</tr>
<tr>
<td>DFL GEN</td>
<td>1.8700/10</td>
<td>44.42</td>
<td>87.85</td>
<td>123.130</td>
<td>256/259</td>
<td>355.535</td>
</tr>
<tr>
<td>YEN JAPAN</td>
<td>1.2732/37</td>
<td>25/27</td>
<td>53/59</td>
<td>82/79</td>
<td>169/165</td>
<td>365.535</td>
</tr>
<tr>
<td>AUD CAN</td>
<td>.7311/16</td>
<td>20/24</td>
<td>50/58</td>
<td>74/72</td>
<td>150/146</td>
<td>318/308</td>
</tr>
</tbody>
</table>

*Key:

STG = Pound sterling
DMK = Danish mark
SFR = Swiss franc
FFR = French franc
BUC = Belgian franc
LIT = Lithuanian litas
CFA = CFA franc
JPN = Japanese yen
AUD = Australian dollar

Note that currencies are quoted as units of foreign currency to dollars, except for sterling, which is quoted in dollars per pound sterling. Prices are for Chase Manhattan Bank European branches and are representative of transaction prices for each location in the represented hour. For example, Italian lira quotes are for the Chase Milan Branch at 10:39 AM in Italian lira.

Source: Reprinted by permission of Reuters.

For FRAs exists in various currencies, the key ones being U.S. dollars, pounds sterling, deutsche marks, Swiss francs, and Japanese yen. However, given the volatility in the dollar and sterling interest rates, these two currencies have the largest volume. Although the market is truly global, most of the business is done in London. Within the sterling and dollar markets, quotes are two-way, with a bid-offer spread rate presented to the customer. This is, of course, similar to any actively traded securities market, and it closely parallels the forward foreign exchange market.
The Contract

Although each institution that deals in forward rate agreements has its own "terms and conditions," the British Bankers Association (BBA) terms and conditions have become the industry standard. Consequently, much of our discussion will refer to these terms and conditions.

Key to a forward rate agreement is the fact that it is a forward contract on interest rates, not a forward commitment to make a loan or take a deposit. Consequently, a key clause of any FRA agreement underscores the point that neither party to the contract has a commitment to lend or to borrow the contract amount (i.e., the principal of the contract). Furthermore, all FRA contracts contain a clause referring to normal banking practice, which commits the parties to specific performance. If the performance commitment is not met, this clause makes the outstanding net cash value of the contract subject to the same conditions that would apply to a loan not performed upon. Such a clause illustrates that: (1) an FRA, like a foreign exchange forward, is essentially a credit instrument; and (2) just as with a foreign exchange forward, no front-end cash transaction takes place; all value is conveyed at maturity.

Dealing on an FRA is done by phone or telex (or by letter). Given the nature of the transaction, most firms stipulate that dealing will be done only if the counterparty allows the firm to tape an oral agreement. However, confirmation must be made in the form of a telex registered letter.

The formula as stated by the BBA for calculating the settlement of a forward contract is:

\[ \frac{(L - R) \times D \times A}{(B \times 100) + (L \times D)} \]

11. Ibid., section D, p. 8.
Forward Contracts

or

(b) when $R$ is higher than $L$

$$
\frac{(R - L) \times D \times A}{(B \times 100) + (L \times D)}
$$

where

$L$ = BBA Interest Settlement Rate
$R$ = Contract Rate
$D$ = Days in Contract Period
$A$ = Contract Amount
$B$ = 360 or 365 days according to market custom

Examining the preceding formula, we can see why it makes sense. One party to the contract agrees with the other on the future interest rate at which they are willing to contract ($R$). Upon settlement—in three months, twelve months, or whatever—the actual rate is observed in the market. That rate is $L$ and is determined as the 11:00 a.m. London rate in the interbank market. Knowing $L$, we know the difference between $L$ and $R$. Depending on whether you were the buyer or the seller, your gain or loss is this difference times the number of days in the contract period times the contract amount, divided by the term ($P \times 100$) (which adjusts the days to the correct basis) plus the appropriate interest rate discounting the original contract ($L \times D$). To obtain an intuitive feel for these calculations, let's consider an example.

Example

Using a FRA

Suppose a party entered into a FRA contract with Citicorp (CITI). Neglecting bid-ask spread considerations, the contracting party wants to be paid (to receive) income on a contract amount of $100 million (U.S.) if, in three months, three-month LIBOR is more than what is expected given today's yield curve (the three-month rate is three months). If the three-month forward rate is 10%, CITI

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12. This formula has or be changed for FRAs of duration greater than twelve months. This formula is modified to create a series of one-year rates for the contract period, which are then discounted appropriately.
would agree to pay if the settlement rate were above 10% and would agree to receive if the rate were below 10%.

Let’s assume that the parties agree to the contract on March 15 for a June 15 spot date for determining the settlement rate, with actual transference of funds occurring two business days later—June 17 (the settlement date). What happens if on June 15 the rate is exactly 10%? Nothing. Neither our contracting party nor CITI receives or pays. However, suppose that June 15 arrives and the three-month LIBOR rate determined by the reference bank in the contract is 11%. CITI must pay on June 17 to the contracting party. How much does CITI pay? Before resorting to the formula, let’s think it through logically. The notional amount is $100 million; the difference in settlement and contract rates is 1.0%; and the contract was for 92 days. First, we need to know how much 92 days of interest differential is worth for $100 million:

$$ (100,000,000) \times (0.01) \times (92/360) = 255,555.56 $$

This is the value after holding another three months; however, the payments are going to be made at settlement, so this amount must be discounted by the three-month LIBOR in effect:

$$ \frac{255,555.56}{1 + [0.11 \times (92/360)]} = 248,568.03 $$

We get precisely the same answer if we utilize the formula provided by the BBA:

$$ \frac{(L - R) \times B \times A}{(B \times 100) + (L \times D)} = \frac{(11.00 - 10.00) \times 92 \times 100,000,000}{36,000 + (11.0 \times 92)} = \frac{9,200,000,000}{37,012} = 248,568.03 $$

The Forward Interest Rate

In the previous discussion we took as given the contract rate—the forward interest rate. The fact is, however, that the forward interest rate, like the forward foreign exchange rate, can be derived from an arbitrage.

13. Logically our inventive approach could be written as:

$$ A \times \frac{\text{LIBOR} \times \frac{B}{L}}{1 + \frac{\text{LIBOR} \times B}{L}} $$

which, when manipulated algebraically, equals the BBA formula.
80  Forward Contracts

condition. We will first illustrate with an example and then generalize
the arguments.

Example

Deriving a forward rate

Suppose you are faced with a tough decision. You have $100 to invest for two
years. Should you invest for one year then take the proceeds and invest for
another, or should you invest for two years? Is there any difference?

From the newspaper you obtain interest rates for one-year and two-year
investments. One-year investments yield 7%, and two-year investments yield
8%. Is the fact that the two-year rate exceeds the one-year rate sufficient
information for you to make your decision?

No. To compare the two investments, you not only need to know the rates
for one and two years, but you also need to know what investment rates you
can expect in one year’s time. You need to know the market expectation of
that one-year reinvestment rate in one year’s time.

How can you calculate the rate expected by the market? That is, how can
you calculate the one-year rate that will prevail one year hence—the forward
rate?14 Remembering the lesson from the forward foreign exchange market that
the foreign exchange forward rate is the one that eliminates arbitrage profit,
you seek an arbitrage condition to guide you. At contracting time, conditions
must be such as to eliminate arbitrage. The forward rate, $f$, must be such that
borrowing or lending for one year and then rolling over the borrowing or lending
for a second year [$100(1 + 0.07) \times (1 + f)]$ is expected to be equivalent to
the borrowing or lending for two years [$100(1 + 0.08)^2$]. Using this arbitrage
relation,

\[
(1 + f) = \frac{100(1 + 0.08)^2}{100(1 + 0.07)}
\]

14. Our discussion of these yield-curve calculations is standard and is covered in many
texts. See, for instance, Richard Brealey and Stewart Myers, Principles of Corporate

15. Alternatively, we can look at the future values of the two strategies. If you invest
for one year and roll over at the end of the first year, the future value is

\[
P_F = 1.07 \times 100 \times 1.07 = 116.64
\]

If you invest for two years, the future value is

\[
P_F = 1.08^2 \times 100 = 116.64
\]

The future value is the same. Of course, if it were not, then the forward rate we calculated
would not be the arbitrage rate, and one strategy would dominate the other.

So \( f = 9.0099\% \).  

The decision to invest for one year and then reinvest for the second year versus investing for the full two years can now be made. The "market" expects the one-year reinvestment rate to be 9.0099\%. If you expect the one-year rate in one year to be higher than 9.0099\%, then invest for one year with plans to reinvest for one year at the end of the first year. If, however, you expect the one-year rate in one year to be below 9.0099\%, then invest for two years today.

To generalize the formula for one-year investments, we recognize that the forward rate is implicit in the yield curve itself. For example, if we wish to know the one-year rate one year from today \((t = 0)\), we solve the equation

\[
(1 + sR_1)(1 + tR_2) = (1 + tR_3)^2
\]

where

- \( sR_1 \) = the one-year interest rate today
- \( sR_2 \) = the two-year interest rate today
- \( tR_3 \) = the forward interest rate for one year between years 1 and 2

If we wish to know the forward rate from year \( j \) to year \( k \), we solve

\[
(1 + sR_j)^{k-j} = (1 + sR_k)^{k-j} 
\]

For periods less than one year, the equation must be modified. For instance, if we want to know the forward rate from month \( j \) to month \( k \) and if the interest rates are quoted as simple rates, we solve for \( R_k \) in

\[
(1 + (j/12)sR_j) \times (1 + [(k-j)/12]R_k) = (1 + (k/12)sR_k)
\]

If, however, the interest rates are quoted compounded, the appropriate form is

\[
(1 + sR_j)^{(k-j)/12} \times (1 + R_k)^{(k-j)/12} = (1 + sR_k)^{k-j/12}
\]

**Bid-Ask Spreads**

Just as foreign exchange forwards grew out of parallel borrowing and depositing, FRAs evolved from forward forward contracts. A forward forward is an obligation in which one financial institution agrees to deposit money at another institution at a specified future date, at a specified interest rate (set at contract origination). Here the credit risk
is obvious; physical deposits are to be made. The FRA was developed to strip the deposit risk from the interest rate risk. With an FRA only the cash flows need be exchanged, so the credit risk is reduced.

The language of the FRA market reflects this evolution. The bid-ask spread is called deposit-borrow spread. On a Reuters screen the FRA rate might be quoted as 11-10\(\frac{1}{2}\). This means you can borrow at 11 or deposit at 10\(\frac{1}{2}\). Of course, since FRAs are forwards, there is no actual borrowing or depositing as would be the case for forward forward contracts.

As with foreign exchange forwards, the deposit-borrow spread is circumscribed by the actual cost associated with the physical depositing and borrowing that would occur as an alternative to using the FRA. If FRAs are priced, then, as if they were actual deposits (and borrowings) for different maturities, but the bid-offer spread is narrower than for actual borrowings and deposits because the credit risk of the transaction has been reduced significantly. There is, as pointed out earlier, still residual credit risk similar to that inherent in a forward foreign exchange rate contract.

Figure 4-5 illustrates the bid-offer spreads inherent in a FRA contract. This screen presents FRAs for three "months" duration commencing in

<table>
<thead>
<tr>
<th>Dates</th>
<th>Dollars</th>
<th>Dates</th>
<th>Dollars</th>
<th>Sterling</th>
</tr>
</thead>
<tbody>
<tr>
<td>3M V 6M</td>
<td>7.93-7.87</td>
<td>3M V 6M</td>
<td>9.96-9.88</td>
<td></td>
</tr>
<tr>
<td>5M V 8M</td>
<td>8.20-8.14</td>
<td>5M V 8M</td>
<td>10.02-9.94</td>
<td></td>
</tr>
<tr>
<td>6M V 9M</td>
<td>8.35-8.29</td>
<td>6M V 9M</td>
<td>10.03-9.95</td>
<td></td>
</tr>
<tr>
<td>9M V 12M</td>
<td>8.49-8.43</td>
<td>9M V 12M</td>
<td>10.04-9.96</td>
<td></td>
</tr>
<tr>
<td>4M V 10M</td>
<td>8.30-8.24</td>
<td>4M V 10M</td>
<td>10.13-10.05</td>
<td></td>
</tr>
<tr>
<td>5M V 11M</td>
<td>8.41-8.35</td>
<td>5M V 11M</td>
<td>10.14-10.06</td>
<td></td>
</tr>
<tr>
<td>6M V 12M</td>
<td>8.49-8.43</td>
<td>6M V 12M</td>
<td>10.12-10.04</td>
<td></td>
</tr>
</tbody>
</table>

Key:
The date 3M V 6M refers to three months' money in 3 months; 4M V 7M refers to three months' money in 4 months and so on.

Source: Reprinted by permission of Reuters.
three months' time, four months' time, etc.; also shown are six-month FRAs. Again, the language used—3 vs. 6 or 5 vs. 8—is a holdover from the forward forward contract days. The indicative rates quoted reflect the forward rates from the zero-coupon LIBOR curves for dollars and sterling.

Example

Booking a FRA

Suppose Banco Lar of Brazil wants to lock in its U.S. dollar LIBOR borrowing costs for three months, six months from now.

Banco Lar’s trader, having established that credit lines exist, calls Barclays Bank in London. Since the yield curve slopes upward, he knows that the quotes will be well above today’s three-month LIBOR. (He has unlearned the yield curve to get the forward interest rate, sR6. But when he calls for a quote, he is not going to let Barclays know whether he is going to deposit or borrow.) The Barclays trader quotes 8.49-8.47 for “$30” ($10,000,000 notional principal). Both parties understand that FRA/BBA terms apply (they have already exchanged master forward agreements). The Banco Lar trader says no; his quote is for “$12.” The Barclays trader quotes 8.49-8.43—exactly the same conditions. (For a very large amount, say, $100,000,000, the bid-offer spread could change.)

The Banco Lar trader takes the 8.49 borrowing rate. Banco Lar has now fixed its borrowing rate—though not the physical borrowing—for a three-month period in six months.