In the preceding chapter, we presented a simplified approach to the valuation of an option, the so-called "binomial method." Although the binomial method is widely used for valuing options, the most well known option pricing model is that proposed by Fischer Black and Myron Scholes in 1973. The Black-Scholes model was the first successful model for valuing options, and it spawned a number of variations.

Indeed, the Black-Scholes approach lies behind the development of the binomial method. The paradigm that makes the binomial model work is the concept of the arbitrage portfolio: options and shares can be combined to form a portfolio that is riskless. This paradigm is the central feature of the Black-Scholes approach. Black and Scholes noted that a riskless hedge can be created out of positions in the option and shares of underlying stock. Because the hedge is (instantaneously) riskless, arbitrage ensures that the return to the hedge is the riskless rate. By combining this equilibrium condition with the appropriate boundary conditions, Black and Scholes were able to derive a specific option pricing model.

1. The Black-Scholes method is based on the assumption that the underlying distribution of asset price at maturity is lognormal and the option is European. It follows that the binomial method is particularly useful when a lognormal distribution is not appropriate for the underlying asset price and/or when the option is American and there is the possibility of an early exercise. As we will indicate in Chapter 16, the binomial model is widely used in valuing interest rate options.

Because the Black-Scholes model is by nature mathematical, this chapter is more analytical than those that precede or follow it. However, keep in mind that our purpose is not to turn you into some kind of rocket scientist but to provide you with some insights into this crucial pricing relation. Hence, in the discussion that follows, we have attempted to hold the mathematics to a minimum. The reader interested in a more mathematical approach is invited to examine the sources provided.

Valuing a European Call Option

Like any other mathematical model, the Black-Scholes model for valuing a European option is based on a set of assumptions. The first three assumptions are straightforward:

1. Transaction costs and taxes are zero, and there are no penalties for short sales.
2. The riskless interest rate is known and constant.
3. The stock pays no dividends.

The fourth assumption is a little more complicated:

4. The market operates continuously, and the stock price follows what is referred to as a continuous Itô process.

For those of you for whom the "continuous Itô process" is foreign, we provide the following aside.

Aside

A continuous Itô process

Were this a mathematics text, we could define an Itô process simply as "a Markov process in continuous time." Since this is not a math text, a little more detail is in order.

A Markov process is one in which the observation in time period \( t \) depends only on the preceding observation. For example, if a stock price follows a Markov process, the stock price \( S \) in period \( t \) could be defined as

\[
S_t = X(S_{t-1}) + E_t,
\]

where \( X \) is a constant and \( E_t \) is a random error term.

A process is continuous if it can be drawn without picking the pen up from the paper.
Combining the preceding conditions, the following figure provides an illustrative path of a random variable $S$ that follows an Ito process through time.

In general, the hedge portfolio—the arbitrage portfolio—is formed by combining both stock and call options. The value of the hedge portfolio, $V_H$, can be expressed as:

$$V_H = Q_S S + Q_C C$$  \hspace{1cm} (15-1)

where $S$ is the price of a share of the stock, $C$ is the price of a European call option to purchase one share of the stock, $Q_S$ is the quantity of stock in the hedge, and $Q_C$ is the quantity of call options in the hedge.

The change in the value of the hedge, that is, the derivative of the value of the hedge, $dV_H$, is simply

$$dV_H = Q_S dS + Q_C dC$$  \hspace{1cm} (15-2)

Note in Equation (15-2) that, since at some point in time the quantities of options and stock are given, the change in the value of the hedge results simply from the change in the prices of the assets, $dS$ and $dC$.

As we have noted, the stock price is assumed to follow a continuous Ito process, so there exists a specific mathematical expression for $dS$. We know that the call price is a function of the stock price and the time remaining to expiration of the option. What we need is a mathematical expression for $dC$. This is provided by Ito’s lemma. As indicated in the following aside, Ito’s lemma provides an expression for the differential of functions of variables that follow an Ito process.
Aside

Ito’s lemma

Ito’s lemma is a differentiation rule for random variables whose movements can be described as an Ito process. If stock price follows a simple Ito process, the returns to the stock can be represented by

$$\frac{dS}{S} = \mu dt + \sigma dZ$$

Where $\mu$ and $\sigma$ are constants, $dt$ is the change in time, and $dZ$ is a normally distributed random variable with a mean of zero and a variance $dt$. Multiplying both sides of the equation by $S$ leads to

$$dS = \mu S dt + \sigma S dZ$$

where the expected value and variance of $dS$ are

$$E[dS] = \mu S dt \quad Var(dS) = \sigma^2 S^2 dt$$

As noted earlier, the value of a call option written on the stock is a function of the stock price and the time remaining to expiration of the option:

$$C = C(S, t)$$

What we want to know is the effect of incremental changes in $S$ and $t$ on the value of the call option, $C(S+\Delta S, t+\Delta t) - C(S,t)$. To obtain $C(S+\Delta S, t+\Delta t)$, we use a second-order Taylor series approximation:

$$C(S+\Delta S, t+\Delta t) \approx C(S,t) + \frac{\partial C}{\partial S} \Delta S + \frac{\partial C}{\partial t} \Delta t + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} (\Delta S)^2$$

Then

$$dC = C(S+\Delta S, t+\Delta t) - C(S,t) = \frac{\partial C}{\partial S} \Delta S + \frac{\partial C}{\partial t} \Delta t + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} (\Delta S)^2$$

Interpreting $(\Delta S)^2$ as the variance of $dS$, we can express $dC$ as

$$dC = \frac{\partial C}{\partial S} \Delta S + \frac{\partial C}{\partial t} \Delta t + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 \Delta S$$

A crucial insight of the preceding analysis is that the change in the call price, $dC$, can be expressed as the sum of two terms, one related to the change in the stock price and the other related to the change in the time to maturity:
Figure 15.1. The Change in the Call Price, \( dC \), from a Change in the Stock Price, \( dS \), is the Slope of the Tangent, \( \frac{\partial C}{\partial S} \), Times the Stock Price Change, \( dS \).

\[
dC = \frac{\partial C}{\partial S} dS + \left( \frac{\partial^2 C}{\partial S^2} S \right) dt
\]  

It is helpful to look at this decomposition of the change in the call price graphically. Figure 15.1 illustrates the first term in Equation (15-3). For small changes in the stock price, the associated change in the call price is given by the slope of the tangent, \( \frac{\partial C}{\partial S} \), times the stock price change, \( dS \). The second term in Equation (15-3), the component related to the change in the time to maturity, is illustrated in Figure 15.2. Given the prevailing stock price, \( S_0 \), a decrease in the time to maturity decreases the present value of the exercise price. Thus, from Equation (15-3), decreasing the time remaining to maturity decreases the value of the call. Note that on the right-hand side of the equation, only the first term, \( (\partial C/\partial S) dS \) is stochastic; the rest of the terms are deterministic.

If Equation (15-3) is substituted into Equation (15-2), we obtain the following expression for the change in the value of the hedge portfolio:

\[
dV_H = Q_0 dS + Q_0 \left( \frac{\partial C}{\partial S} dS + \left( \frac{\partial^2 C}{\partial S^2} S \right) dt \right)
\]  

(15-4)
Figure 15-2. The Change in the Value of the Call, $dC$, from a Change in the Time to Maturity, $dt$, is the Shift in the Curve When the Present Value of the Exercise Price Changes from $e^{-rT} X$ to $e^{-rT} X$. 

\[ \frac{dc}{dt} = \frac{1}{S_0} \frac{1}{2} \sigma^2 S_0^2 \rho dt \]

If the quantities of stock and of call options in the hedge portfolio are chosen so that $Q_s/Q_C$ equals $-\frac{\partial C}{\partial S}$, the first two terms on the right-hand side of Equation (15-4) sum to zero. And, since these are the only stochastic terms, it follows that if $Q_s/Q_C$ is equal to $-\frac{\partial C}{\partial S}$, the change in the value of the hedge becomes deterministic; this is, the hedge portfolio becomes riskless.

This means that, with the appropriate long position in the stock and short position in the call, an increase in the price of the stock will be offset by the decrease in the value of the short position in the call, and vice versa. This can be illustrated graphically by returning to Figures 15-1 and 15-2. By setting $Q_s/Q_C$ equal to $-\frac{\partial C}{\partial S}$ the unanticipated change in the call price due to stock price change (illustrated in Figure 15-1) is hedged by the stock price change itself, so that the predictable

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3. The riskless hedge could also be created with a short position in the stock and a long position in the call. Note that the restriction is placed on the ratio $Q_s/Q_C$; it makes no difference which asset is short.
change in the call price from the reduction in the time to maturity, illustrated in Figure 15-2, is all that remains.

Hence, the insight provided by Black and Scholes is that, if the quantities of the stock and of the call option in the hedge portfolio are continuously adjusted in the appropriate manner as asset prices change over time, then the return to the portfolio becomes riskless. Setting $Q_C = -1$ and $Q_S = \left( \frac{\partial C}{\partial S} \right)$ in Equation (15-4) yields

$$dV = \left( \frac{\partial C}{\partial t} + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 \right) dt \quad (15-5)$$

Thus, we have mathematically eliminated all the stochastic terms (since $dt$ is deterministic, $dV$ is deterministic), so this hedge portfolio is riskless. Hence, the return to the hedge portfolio must equal the riskless rate:

$$\frac{dV}{V} = (r) dt \quad (15-6)$$

We are now ready to do some arithmetic to derive an explicit expression for the change in the call price. Imposing $Q_C = -1$ and $Q_S = \left( \frac{\partial C}{\partial S} \right)$ on Equation (15-1),

$$V = \left( \frac{\partial C}{\partial S} \right) S - C \quad (15-7)$$

Then, using Equation (15-7) in Equation (15-6),

$$dV = \left( \frac{\partial C}{\partial S} \right) dt - (rC) dt \quad (15-8)$$

and setting the right-hand sides of Equations (15-5) and (15-8) equal to one another, we obtain

$$\frac{\partial C}{\partial t} = r C - \frac{\partial C}{\partial S} S - \frac{1}{2} \frac{\partial^2 C}{\partial S^2} (S^2 \sigma^2) \quad (15-9)$$

We are now close to our objective: an expression for the value of the call. We have in Equation (15-9) an expression for the change in the value of the call—what mathematicians call a differential equation. We need to get from the differential equation to an equation for the value of the call; that is, given Equation (15-9), we want to solve for the value of the call.
Aside

Differential equations

A differential equation is simply an equation that contains derivatives. If there is a single independent variable, the derivatives are ordinary derivatives and the equation is an ordinary differential equation. For example, an ordinary differential equation would be \( \frac{dy}{dx} = 0.8 \). If there are two or more independent variables, the derivatives are partial derivatives and the equation is called a partial differential equation. Note that Equation (15-9) is a partial differential equation since it involves both \( \partial C/\partial S \) and \( \partial C/\partial t \).

To gain some intuition, consider again the simple ordinary differential equation above, \( \frac{dy}{dx} = 0.8 \). Since the differential equation is equal to a constant, 0.8, the equation is telling us that the slope of \( y \) plotted against \( x \) would be a constant 0.8. In other words, this differential equation implies that the function linking \( y \) and \( x \) is a straight line with a slope of 0.8. But there are an infinite number of straight lines with a slope of 0.8—which one is the correct one? To identify a single line, we also need a “boundary condition,” a fixed point to lie down the function. Hence, also knowing that if \( x = 0, y = 2 \) tells us that the unique solution we seek is \( y = 2 + 0.8x \).

As noted in the preceding aside, to derive an expression for the call value, we must have a boundary condition, something to tie down our expression for the change in call value. The required boundary condition for the solution of this differential equation is the condition we outlined in Chapter 15. At expiration of the option, the option value must equal the maximum of either the difference between the stock price and the exercise price, \( S^* - X \), or zero:

\[
C^* = \max(S^* - X, 0) \quad (15-10)
\]

Thus, the value of the call option will be obtained by solving Equation (15-9) subject to Equation (15-10).

Before proceeding to the solution to our problem, we should note that whatever the form of the solution, it must be a function only of five

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4. In general, for the solution of a partial differential equation (a differential equation that is a function of more than one variable), one boundary condition is required for each dimension. Equation (15-10) is the boundary condition in the time dimension. In the stock price dimension, the boundary condition is that the call price is zero if the stock price is zero. However, because it is explicitly assumed that the call price is exponentially throughput, the stock price cannot be zero. Therefore the boundary condition will never be binding and, in this special case, can be ignored.
variables: the stock price, \( S \); the exercise price, \( X \); the variance rate, \( \sigma^2 \); time, \( t \); and the riskless interest rate, \( r \). This is because these are the only variables that appear in the problem.

To obtain the solution to the differential equation Black and Scholes noted that Equation (15-9) could be transformed into an equation that is familiar to physicists: the "heat exchange equation." However, since we anticipate that few readers are familiar with the heat exchange equation, a more intuitive solution technique is likely to be more useful and informative.

Note that when we described the equilibrium return to the hedge portfolio, the only assumption we made about the preferences of the market participants is that two assets that are perfect substitutes must earn the same rate of return; because the hedge portfolio has zero risk, it must earn the riskless rate of return. Hence, the pricing model implied by Equation (15-9) must be invariant to preferences since no assumptions involving the risk preferences of the economic agents have been made. It follows, then, that if we can find a solution to the problem for a particular preference structure, it must also be the solution to the differential equation for any other preference structure that permits a solution.

Therefore, to solve Equation (15-9), we choose the preference structure that most simplifies the mathematics: We assume a preference structure where all agents are risk-neutral. In a risk-neutral world, the expected rate of return on all assets is equal. Hence, the current call price is the present value of the expected call price at expiration of the contract, \( E[C^*] \), discounted by the market-wide discount rate, \( r \). That is,

\[
C = e^{-rT}E[C^*] \tag{15-11}
\]

where \( T \) is the amount of time remaining until expiration. If we assume further that the distribution of stock prices at any future date will be lognormal, Equation (15-11) can be expressed as

\[
C = e^{-rT} \int_0^\infty (S^* - X)L'(S^*)dS^* \tag{15-12}
\]

where \( L'(S^*) \) is the lognormal density function.

Equation (15-12) is integrated using a theorem from Smith. The result of this integration is the Black-Scholes solution to the European call pricing problem:

\[
C = S \times N\left( \frac{\ln(S/X) + (r - \sigma^2/2)T}{\sigma \sqrt{T}} \right) - e^{-rT} \times N\left( \frac{\ln(S/X) + (r - \sigma^2/2)T}{\sigma \sqrt{T}} \right)
\]  

(15-13)

where \(N(\cdot)\) denotes the cumulative standard normal. The payoff function for the call option and the lognormal density function for the stock prices are presented in Figure 15-3. Figure 15-4 illustrates graphically the relation between the Black-Scholes valuation of the call and the stock price, given the exercise price, the time to maturity, and the riskless rate.

**Interpreting the Black-Scholes Pricing Model**

As we anticipated, the solution to Equation (15-9)—the Black-Scholes option pricing model—involves only five variables:

\[
C = C(S, X, T, r, \sigma)
\]

(15-14)

In Equation (15-14), the signs above the variables represent their partial derivatives. The partial effects again have intuitive interpretations:

- As the stock price increases, the expected payoff of the option increases.
- With a higher exercise price, the expected payoff decreases.
- With a longer time to maturity, the present value of the exercise payment is lower, thus increasing the value of the option.

Figure 15-3. (a) Dollar Payoff to Call as a Function of Stock Price, \( C' = \max\{0, S' - X\} \), (b) Lognormal Density Function of Stock Prices at \( t', L'(S') \).

- With a higher interest rate, the present value of the exercise payment is lower, thus increasing the value of the option.
- With a larger variance for the underlying stock price (or with a longer time to maturity), the probability of a large stock price change during the life of the option is greater. Since the call
price cannot be negative, a large range of possible stock prices increases the maximum value of the option without lowering the minimum value.

Additional understanding of the Black-Scholes model is obtained by going a little deeper into the risk-neutral pricing outlined in Equations (15-11) and (15-12). With risk neutrality, Equation (15-12) can be rewritten to express the value of the call in terms of conditional expected values:

\[ C = e^{-rT} \mathbb{E}(S_\tau | S_\tau > X) \text{prob}(S_\tau > X) - e^{-rT} X \text{prob}(S_\tau > X) \]  

(15-15)

The two terms in Equation (15-15) have natural interpretations: The first term is the product of (1) the discounted expected value of the stock price at contract maturity, conditional on the terminal stock price exceeding the exercise price, and (2) the probability that the stock price at contract maturity is greater than the exercise price. The second term is the product
of (1) the discounted exercise price and (2) the probability that the stock price at contract maturity exceeds the exercise price.

Let's examine the Black-Scholes solution in Equation (15-13), by considering two extreme cases:

1. An extremely out-of-the-money call ($S_X << X$). For the extremely out-of-the-money call, the ratio of stock price to exercise price is significantly less than 1: $S_X << 1$. Thus, the natural logarithm of that ratio is negative—\(\ln(S_X/X) < 0\)—so, the area under a standard normal curve from negative infinity to that point is very small: \(N(\ln(S_X/X)) \to 0\). Therefore, the value of an extremely out-of-the-money call is approximately zero.

2. An extremely in-the-money call ($S_X >> X$). For the extremely in-the-money call, the ratio of stock to exercise price is significantly greater than 1: $S_X >> 1$. Thus, the natural logarithm of that ratio is positive—\(\ln(S_X/X) > 0\)—so the area under a standard normal from negative infinity to that point is close to one, \(N(\ln(S_X/X)) \to 1\). Therefore, the value of an extremely in-the-money call is approximately \(S - e^{-rT} X\).

The derivative of the Black-Scholes call price with respect to change in the stock price—the option's delta—is the first cumulative standard normal term:

\[
\frac{\partial C}{\partial S} = N \left( \frac{\ln(S/X) + (r + \sigma^2/2)T}{\sigma \sqrt{T}} \right) \tag{15-16}
\]

For an out-of-the-money call, the delta is virtually 0. For an extremely in-the-money call, the delta is virtually 1. And, for an at-the-money call, the delta is approximately \(\frac{1}{2}\).

Aside

Fraternity Row

Understanding the changes in the value of an option, which result from changes in the price of the underlying asset, as the time to expiration, and in the volatility of the asset price, is essential in managing a portfolio of options. On the street, these derivatives have been given Greek (or pseudo-Greek) names.
Delta is \(\partial C/\partial S\), the expected change in the option premium for a small change in the price of the underlying asset, other variables constant. Delta varies from 0 to 1 for long calls and from 0 to -1 for long puts.

Gamma is \(\partial^2 C/\partial S^2\), the expected change in delta for a small change in the price of the underlying asset, other variables constant. Gamma measures the convexity of the option pricing function, or the stability of delta. (If delta is like velocity, then gamma is like acceleration.) Because convexity benefits long option holders, long-option gammas are positive and short-option gammas are negative.

Theta is \(\partial C/\partial T\), the expected change in the option premium for a small change in time to expiration, other variables constant. The theta of a long option is negative.

Vega is \(\partial C/\partial \sigma\), the expected change in the option premium for a small change in volatility, other variables constant. (We presume that this term was coined because there is no Greek letter beginning with V—for volatility.) Vegas of long positions are positive.

Empowering the Formula: Implied Volatilities

The Black-Scholes option pricing model of equation (15.13) involves only five variables: the asset price, the exercise price, time to maturity, the riskless rate, and the volatility of the asset price. In applying the formula, the first four values are simple to obtain. The exercise price and the maturity date are stated in the contract, the asset price is quoted in the market, and the riskless rate can be derived from quoted bond prices. It is volatility that causes the difficulties, since this variable is not directly observable. If it can be assumed that the historical volatility provides an accurate indicator of future volatility, then the simple standard deviation of the log of the rate of return, \(\sigma\), can be employed to calculate the call price.

Another method is to recognize that for lived options, the market's assessment of the value of the call is available as well as stock prices and interest rates. With that information, one can calculate the implied volatility, which, when plugged into the Black-Scholes formula along with the current stock price, interest rate, exercise price, and time to maturity, yields the observed call price.

In calculating implied volatilities, a few caveats are in order. First, the average variance rate expected over the next one month is not necessarily the same as that over the next ten months; there can be a term structure of volatility just as there is a term structure of interest
rates. Therefore, when pricing a given option, employ a measure of implied volatility obtained from other options of comparable maturity. Second, the Black-Scholes model assumes no transaction costs yet bid-ask spreads in options markets can be substantial. To reduce the impact of this problem, use the midpoint of the bid-ask spread for call, stock, and interest rate inputs in calculating implied volatilities. Third, any error in the Black-Scholes pricing model will be translated into the implied volatility. For example, if the stock is expected to pay a dividend prior to the maturity of the option, there is a substantial probability of early exercise.

Relaxing the Black-Scholes Assumptions

Although restrictive assumptions have been employed in this derivation, there has been much work done concerning the effect of their relaxation. Generally, the model seems quite robust to relaxing the basic assumptions.

Restrictions on Short Sales

There are restrictions on the ability of some trades to engage in short sales. For example, there are restrictions on the use of short sale proceeds, as well as the "up-tick" rule (which requires short sales to be executed only after a price increase). Some have worried that this constrains individuals from establishing a riskless hedge with a long position in the call and a short position in the stock. However, if the individual begins with a well-diversified portfolio, the economic effect of the addition of a short position in the stock can be alternatively achieved by reducing an existing long stock position (rather than selling short), thus avoiding the restrictions.

Variable Interest Rates

If interest rates are variable, the basic results of the option pricing model are unaffected as long as zero-coupon riskless bonds of the same maturity as the call are used in establishing the hedge. However, the volatility of the call price will now have two components, one from stock price volatility (\( \sigma^S \)) and the other from bond price volatility (\( \sigma^B \)). By employing comparable-maturity T-bills to hedge, the interest rate
uncertainty cancels out over the life of the hedge, since we know the bond price goes to par at maturity. The price of the option becomes:

\[
C = S \times N \left\{ \frac{\ln(S/X) - \ln(B(T)) + (\sigma^2/2)T}{\sigma \sqrt{T}} \right\} - B(T) \times N \left\{ \frac{\ln(S/X) - \ln(B(T)) - (\sigma^2/2)T}{\sigma \sqrt{T}} \right\}
\]

(15-17)

where \( B(T) \) is the price of a default risk-free bond that matures in \( T \) periods and pays $1.

Dividend Payments

As noted in Chapter 14, since the option holder only has a claim on the capital gain component of the stock return, higher expected dividends over the life of the option reduce the value of the call. Assuming that the dividend is paid continuously and that the dividend yield, \( \delta = D/S \), is constant, Robert C. Merton showed that the pricing equation for a European call on a stock with continuous proportional dividends is:

\[
C = e^{-\delta T} S \times N \left\{ \frac{\ln(S/X) + [r - \delta + (\sigma^2/2)T]}{\sigma \sqrt{T}} \right\} - e^{-\delta T} B(T) \times N \left\{ \frac{\ln(S/X) + [r - \delta - (\sigma^2/2)T]}{\sigma \sqrt{T}} \right\}
\]

(15-18)

For American calls, the valuation of call options on dividend-paying stocks is more complicated, since there is a positive probability of premature exercise of such calls.9

**Discontinuous Share Price: Pricing with Jumps**

In Chapter 14 we saw that if the stock price can take only one of two values at each point, we can derive the binomial pricing model. Here we have seen that with a lognormal diffusion process, we can obtain the Black-Scholes price options. But what if the stock price contains both a diffusion component and a jump component? Robert C. Merton9 examines this case and demonstrates that, in general, no riskless hedge can be formed that simultaneously hedges against the two components of the price change. However, if the jumps are uncorrelated across securities, then jump risk is unsystematic; so jump risk is diversifiable and a security’s expected return is determined by its nondiversifiable risk. If the jumps are also lognormally distributed, Merton shows that the option price will be a weighted average of Black-Scholes solutions, conditional on the number of jumps.

<table>
<thead>
<tr>
<th>Theory Indicates that a Firm is More Likely to Hedge</th>
<th>Hypothesis Supported?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Due to risk-averse owners if: the firm is owned by ill-diversified investors</td>
<td>Yes*</td>
</tr>
<tr>
<td>To reduce expected taxes if: the firm has more tax loss carryforwards</td>
<td>No*</td>
</tr>
<tr>
<td>the firm has more investment tax credits</td>
<td>Yes*</td>
</tr>
<tr>
<td>none of the range of the firm’s pretax income is in the progressive region of the tax schedule</td>
<td>Yes*</td>
</tr>
<tr>
<td>To reduce expected financial distress cost if: The probability of financial distress is higher, that is, if: the ratio of fixed claims to cash inflows is higher</td>
<td>Yes*</td>
</tr>
<tr>
<td>The costs of financial distress are higher, that is, if: the firm is smaller the firm produces a crence good</td>
<td>No*</td>
</tr>
<tr>
<td>To reduce agency costs if: the firm has a larger debt/equity ratio the firm has available a wider range of potential investment projects</td>
<td>Yes*</td>
</tr>
</tbody>
</table>

* Hypothesis not examined
* Result deemed statistically significant by researcher

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