SIGNIFICANCE OF LOSS TANGENT

- $\tan \theta = \frac{\sigma}{\omega \varepsilon}$ is called the loss tangent of the medium.
- It is an important term in the calculations of $\alpha$, $\beta$, $|\eta|$, and $\phi_\eta$.
- May be used to determine how lossy a medium is (border line between lossy dielectric and conductor).
- Loss tangent is also a function of frequency
  - the characteristics of any material are frequency-dependent.
  - At different frequencies, a material can be classified into different types (a good conductor at low frequencies, but a good dielectric at high frequencies). → dissipative medium
\[
\langle \vec{S} \rangle = \frac{1}{2} \text{Re}\left\{\vec{E}_s \times \vec{H}_s^*\right\}
\]

\[
= \frac{1}{2} \text{Re}\left\{E_0 e^{-\alpha z} e^{-j\beta z} \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ E_0 e^{-\alpha z} e^{-j\beta z} 0 0 \\ 0 \frac{E_0}{|\eta|} e^{-\alpha z} e^{j\phi_\eta} e^{j\phi_\eta} 0 \end{bmatrix}\right\}
\]

\[
= \frac{1}{2} \text{Re}\left\{\frac{E_0^2}{|\eta|} e^{-2\alpha z} e^{j\phi_\eta} \hat{z}\right\}
\]

\[
= \frac{1}{2} \frac{E_0^2}{|\eta|} e^{-2\alpha z} \cos \phi_\eta \hat{z}
\]
PLANE WAVES IN LOSSLESS DIELECTRICS

• \( \sigma = 0 \) \( \rightarrow \) \( \varepsilon = \varepsilon - j \frac{\sigma}{\omega} = \varepsilon = \varepsilon_r \varepsilon_0 \) and \( \tan \theta = \frac{\sigma}{\omega \varepsilon} = 0 \)

\( \rightarrow \) The medium is considered a perfect dielectric

• \( \gamma \) is purely imaginary \( \rightarrow \) no attenuation (\( \alpha = 0 \))

\( \rightarrow \) The medium is considered a lossless dielectric

• EM waves propagating in such a material behave very much like waves in free space except for a change in propagation velocity, i.e.,

\[
\eta = \sqrt{\mu / \varepsilon} = \sqrt{\frac{\mu_r \mu_0 \varepsilon_r \varepsilon_0}{\varepsilon_r \varepsilon_0}} = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}}
\]

\( \eta \) is real \( \rightarrow \) \( \vec{E} \) and \( \vec{H} \) are in phase
PLANE WAVES IN LOSSY DIELECTRICS

- If \( \frac{\sigma}{\omega \varepsilon} \ll 1 \) (i.e., less than 0.01), the material is said to be slightly conducting (\( \sigma \) is small).

\[
\alpha \approx \omega \sqrt{\frac{\mu \varepsilon}{2}} \left\{ 1 + \frac{1}{2} \left( \frac{\sigma}{\omega \varepsilon} \right)^2 - 1 \right\} \approx \frac{\sigma}{2} \sqrt{\frac{\mu}{\varepsilon}} \quad \text{and} \quad \beta \approx \omega \sqrt{\mu \varepsilon}
\]

- \( \eta \approx \sqrt{\frac{\mu}{\varepsilon}} \equiv \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}} \) and \( u_p = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} < c \)

- Penetration depth \( (d_p) \) is defined as the distance the wave travels until its amplitude decays to \( e^{-1} \) or 0.368 of its original value.

\[
d_p = \frac{1}{\alpha} = \frac{2}{\sigma} \sqrt{\frac{\varepsilon}{\mu}}
\]
PLANE WAVES IN GOOD CONDUCTORS

• If \( \frac{\sigma}{\omega \varepsilon} \gg 1 \) (i.e., greater than 100), the material is said to be highly conducting \((\sigma \approx \infty, \varepsilon_r = 1, \mu_r > 1)\).

\[
\alpha = \beta \approx \sqrt{\frac{\omega^2 \mu \varepsilon \sigma}{2 \omega \varepsilon}} \approx \sqrt{\frac{\omega \mu \sigma}{2}} = \sqrt{\pi \mu f \mu \sigma} \quad \text{and} \quad d_p = \sqrt{\frac{2}{\omega \mu \sigma}}
\]

• It is noted that \( d_p \) is inversely proportional to wave frequency

\( \rightarrow d_p \) is small when \( f \) is high \( \rightarrow d_p = \delta = \text{skin depth} \)

\( \rightarrow \) most of the conduction current flows on the surface of the conductor, known as skin effect.

• \( u_p = \frac{\omega}{\beta} = \sqrt{\frac{2 \omega}{\mu \sigma}} \quad \text{and} \quad \eta \approx \sqrt{\frac{\omega \mu}{\sigma}} \angle 45^\circ \)
• As $\sigma \to \infty$, the material is considered a perfect conductor.

• $\frac{\sigma}{\omega \varepsilon} \to \infty$ and skin depth becomes infinitesimally small ($\delta \to 0$)

• To keep the conduction current ($\vec{J}_{cd} = \sigma \vec{E}$) finite as $\sigma \to \infty$, $\vec{E} \to 0$ inside the conductor.

$\rightarrow$ EM wave cannot exist inside a conductor.
Ex. For ice, $\sigma \approx 10^{-6} \text{S/m}$, $\varepsilon = 3.2\varepsilon_0$, and $\mu_r = 1$. Find the penetration depth in the MHz range.
Ex. Seawater can be characterized by $\sigma = 4 \text{ S/m}$, $\varepsilon = 81\varepsilon_0$, and $\mu_r = 1$. Classify seawater at frequencies of 60 Hz, 1 MHz, and 100 MHz.
Ex. A 100 Hz EM wave is propagating vertically down into seawater. The electric field just beneath the surface is 1 V/m. What is the intensity of the electric field at a depth of 100 m? Also, compute the radiation power of the wave.
Ex. To shield a room from radio wave interference, the room must be enclosed in a layer of copper five skin depths thick. If the freq to be shielded against is 10 kHz to 100 MHz, what should be the thickness be?
Ex. The $\vec{E}$ field of a linearly polarized uniform plane wave propagating in the $\hat{z}$ direction in seawater ($\sigma = 4$ S/m, $\varepsilon_r = 72$, and $\mu_r = 1$) is $\vec{E}(z,t) = 100 \cos(\pi \times 10^7 t) \hat{x}$ V/m. at $z = 0$.

(a) Determine $\alpha$, $\beta$, $\eta$, $u_p$, $\lambda$, and $d_p$.

(b) Find the distance at which the amplitude of $\vec{E}$ is 1% of its value at $z = 0$.

(c) Write the expressions for $\vec{E}(z,t)$ and $\vec{H}(z,t)$ at $z = 0.8$m.
Ex. A lossy dielectric has $\eta = 200 \angle 30^\circ \Omega$ at a particular frequency.

If $\vec{H}(x,t) = 10e^{-\alpha x} \cos\left(\omega t - \frac{x}{2}\right)\hat{y} \text{ A/m}$, find $\vec{E}$, $\alpha$, $\delta$, and the direction of propagation.