Charles-Augustin de Coulomb

French physicist

14 June 1736 – 23 August 1806
**COULOMB’S LAW**

- An experimental law dealing with the force a point charge exerts on another point charge in material medium

- Expressed mathematically,
  \[
  \vec{F} = \frac{Q_1Q_2}{4\pi\varepsilon R^2} \hat{R} = \frac{Q_1Q_2}{4\pi\varepsilon_0 R^2} \hat{R}
  \]

- directly proportional to the product \( Q_1Q_2 \)

- inversely proportional to the square of the distance along the line joining the two charges

- \( \vec{F}_{12} \) is the force at \( Q_2 \) due to \( Q_1 \)

- \( \vec{F}_{12} = -\vec{F}_{21} = \frac{Q_1Q_2(r_2 - r_1)}{4\pi\varepsilon_0 R^3} \)
RANDOM DISTRIBUTION OF POINT CHARGES

- For N point charges, $Q_1, Q_2, \ldots, Q_N$, randomly located at points with position vectors $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N$, respectively, the total force exerting on $Q$ locate at $\vec{r}$ due to $Q_1, Q_2, \ldots, Q_N$ is

$$\vec{F} = \frac{Q}{4\pi\varepsilon_r\varepsilon_0} \sum_{k=1}^{N} \frac{Q_k (\vec{r} - \vec{r}_k)}{|\vec{r} - \vec{r}_k|^3}$$

$$\vec{E} = \frac{\vec{F}}{Q} = \frac{1}{4\pi\varepsilon_r\varepsilon_0} \sum_{k=1}^{N} \frac{Q_k (\vec{r} - \vec{r}_k)}{|\vec{r} - \vec{r}_k|^3} = \text{Electric Field Intensity}$$

- Electric Field $= \text{Field of electric force per unit charge}$
- $\vec{E}$ is in the same direction as $\vec{F}$
- Unit of $\vec{E}$ is Newton/coulomb (N/C) or Volt/meter (V/m)
Ex. Point charges 1 mC and -2 mC are located at (3, 2, -1) and (-1, -1, 4), respectively. Calculate the electric force on a 10-nC charge located at (0, 3, 1) and the electric field intensity at that point.
NONRANDOM CONTINUOUS DISTRIBUTION OF CHARGES

1. Line charge: line charge density in C/m, $\rho_L$
2. Surface charge: surface charge density in C/m$^2$, $\rho_S$
3. Volume charge: volume charge density in C/m$^3$, $\rho_v$
Ex. Determine the total charge

(a) on the line $0 < x < 5 \text{ m}$ if $\rho_L = 12x^2 \text{ mC/m}$

(b) on the side surface of the cylinder $\rho = 3$, $0 < z < 4 \text{ m}$ if $\rho_S = \rho z^2 \text{ nC/m}^2$

(c) within the sphere $r = 4 \text{ m}$ if $\rho_v = \frac{10}{r \sin \theta} \text{ C/m}^3$
ELECTRIC FIELD CALCULATION

By replacing Q with charge element dQ and integrating,

\[ \vec{E} = \int \frac{\rho_L dl}{4\pi\varepsilon R^2} \hat{R} \]  
(line charge)

\[ \vec{E} = \int \frac{\rho_S dS}{4\pi\varepsilon R^2} \hat{R} \]  
(surface charge)

\[ \vec{E} = \int \frac{\rho_v dv}{4\pi\varepsilon R^2} \hat{R} \]  
(volume charge)