Homework Assignment 2 Due date: January 27th, 2010

1. (a) Convert points P(1, 3, 5), T(0, -4, 3), and S(-3, -4, -10) from Cartesian to cylindrical and spherical coordinates.

(b) Transform vector \( \vec{Q} = \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}} \hat{x} - \frac{yz}{\sqrt{x^2 + y^2 + z^2}} \hat{z} \) to cylindrical and spherical coordinates.

(c) Evaluate \( \vec{Q} \) at T in the three coordinate systems.

2. Calculate the distance between the following pairs of points:
   (a) (2, 1, 5) and (6, -1, 2)   (b) (3, \( \pi /2 \), -1) and (5, 3\( \pi /2 \), 5)
   (c) (10, \( \pi /4 \), 3\( \pi /4 \)) and (5, \( \pi /6 \), 7\( \pi /4 \)).

3. A vector field in mixed coordinate variables is given by \( \vec{G} = \frac{x \cos \phi}{\rho} \hat{x} + \frac{2yz}{\rho^2} \hat{y} + \left(1 - \frac{x^2}{\rho^2}\right) \hat{z} \).
   Express \( \vec{G} \) completely in spherical coordinate system.

4. Given the vector field \( \vec{H} = \rho z \cos \phi \hat{\rho} + e^{-z} \sin \frac{\phi}{2} \hat{\phi} + \rho^2 \hat{z} \) at point (1, \( \pi /3 \), 0), find:
   (a) \( \vec{H} \cdot \hat{x} \);
   (b) \( \vec{H} \times \hat{\theta} \);
   (c) the vector component of \( \vec{H} \) normal to surface \( \rho = 1 \);
   (d) the scalar component of \( \vec{H} \) tangential to the plane \( z = 0 \).

5. If \( \vec{A} = 3\hat{r} + 2\hat{\phi} - 6\hat{\theta} \) and \( \vec{B} = 4\hat{r} + 3\hat{\phi} \), determine
   (a) \( \vec{A} \cdot \vec{B} \)
   (b) \( |\vec{A} \times \vec{B}| \)
   (c) the vector component of \( \vec{A} \) along \( \hat{z} \) at \( \left(1, \frac{\pi}{3}, \frac{5\pi}{4}\right) \)

6. Given vectors \( \vec{A} = 2\hat{x} + 4\hat{y} + 10\hat{z} \) and \( \vec{B} = -5\hat{\rho} + \hat{\phi} - 3\hat{z} \), find
   (a) \( \vec{A} + \vec{B} \) at P(0, 2, -5)
   (b) the angle between \( \vec{A} \) and \( \vec{B} \) at P.

Reading Assignment: Chapter 2