TRANSFORMER

ENERGY IN A COUPLED CIRCUIT

- The total energy *w* stored in a mutually coupled inductor is

$$w = \frac{1}{2}L_1 i_1^{\ 2} + \frac{1}{2}L_2 i_2^{\ 2} \pm M i_1 i_2$$

Positive sign is selected if both currents ENTER or LEAVE the dotted terminals. Otherwise we use Negative sign.

- The energy stored in the circuit cannot be negative because the citcuit is passive.

COUPLING COEFFICIENT

- A measure of the magnetic coupling between two coils.
- It establishes the relationship between mutual inductance and the self-inductances of the two coil:

$$k = \frac{M}{\sqrt{L_1 L_2}} \quad \text{or} \quad M = k\sqrt{L_1 L_2}$$

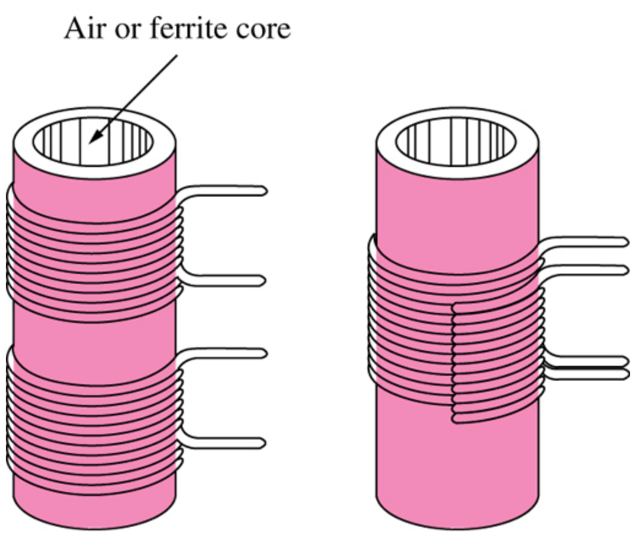
- Since $M \leq \sqrt{L_1 L_2}$ due to the energy consideration, $0 \leq k \leq 1$.

$k = 1$ Perfect Coupling

$k < 0.5$ Loosly Coupling

$k > 0.5$ Tightly Coupling

- Coupling efficiency depends on the closeness of the two coils, their orientation, and their windings.

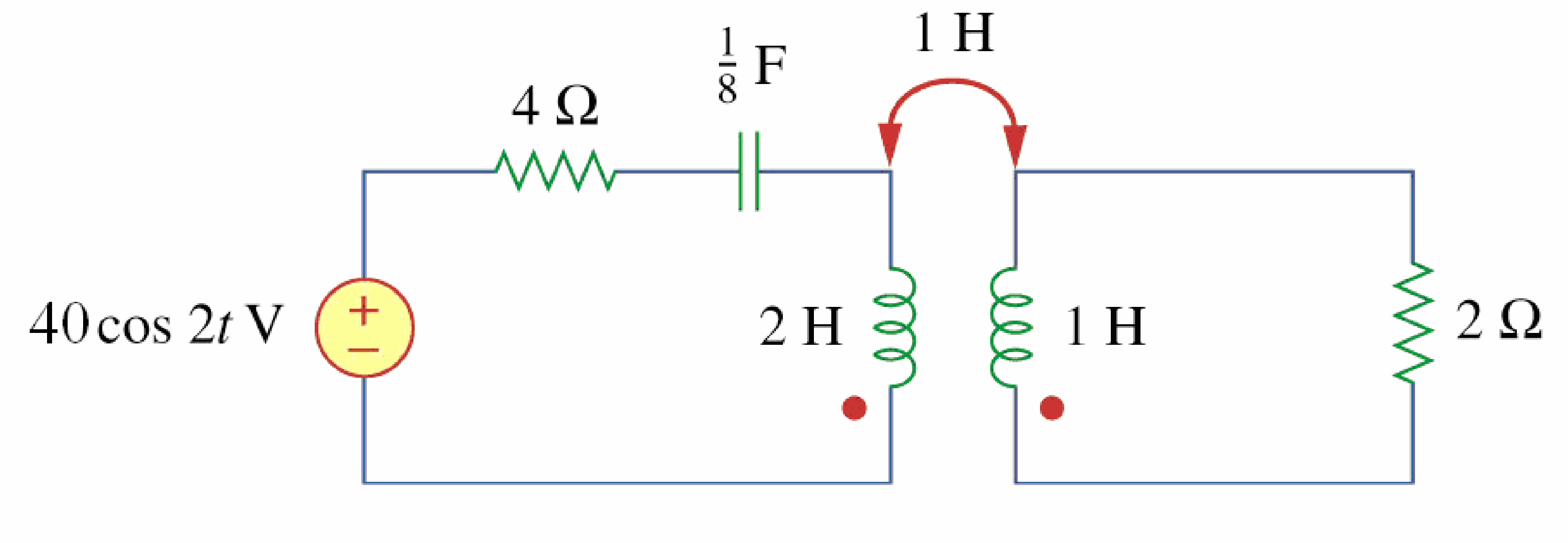


(a) Loosely coupled coil b) Tightly coupled coil

- Air-core coils are loosely coupled.
- Iron-core are tightly coupled.

Ex. Practice Problem 13.3

For the circuit shown, determine the coupling coefficient and the energy stored in the coupled inductors at $t = 1.5$ s.

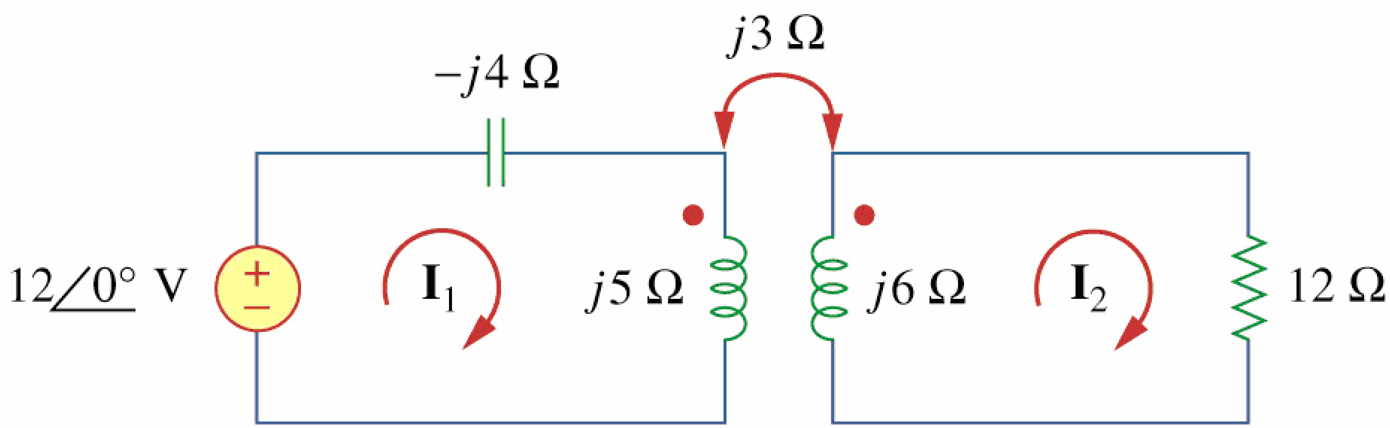


TRANSFORMERS

- A transformer is generally a four-terminal device comprising two or more magnetically coupled coils placed so that the charging flux developed by one will link the others.
- The coil to which the source is applied is called the **primary coil.**
- The coil to which the load is applied is called the **secondary coil.**
- Three basic operations of a transformer are:
  - Step up/down
  - Impedance matching
  - Isolation

LINEAR TRANSFORMERS

- The transformer is called LINEAR if the coils are wound on magnetically linear material (air, plastic, Bakelite, wood, etc.)
- Flux is proportional to current in the windings.
- Resistances R1 and R2 account for losses in the coils.
- Mostly used in radio and TV sets.

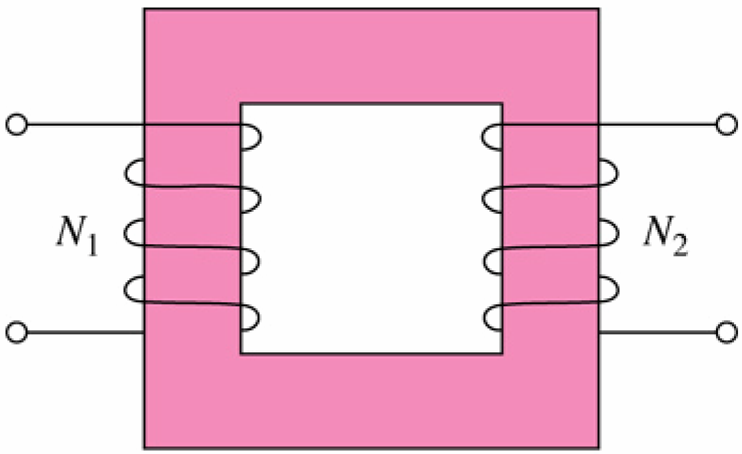


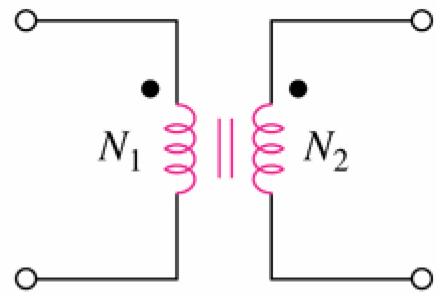

JT-11SSP-8M
jensen
JT-69-HMX
MADE IN U.S.A.
7224
JENSEN
JT-123-SLPC
7289
JENSEN
JT-11-EL
7351
JENSEN
JT-6110K-B
7244
MADE IN USA
JENSEN
JT-DB-E
7346
MADE IN USA
JT-112-LPC
JENSEN
JT-115K-MAX
7358
jensen
JT-10KB-DPC
MADE IN U.S.A.
7106
jensen
JT-110K-HPC
MADE IN U.S.A.
JENSEN
JT-11-FLPCH
7379
MADE IN USA

IDEAL TRANSFORMERS

- An ideal Transformer is a unity-coupled, lossless transformer in which the primary and secondary coils have infinite self inductances.

- A Transformer is ideal if
  1. Large reactance coils;
  2. Unity Coupling k=1.
  3. Coils are lossless (R1=R2=0)

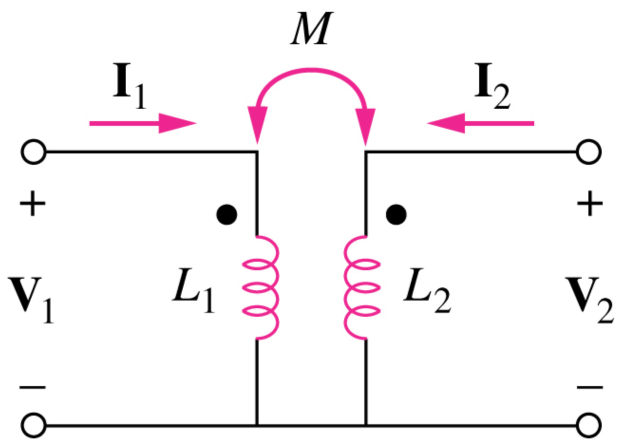




- The input and output voltages and currents of an ideal transformer are related only by the turns ratio.

$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$

$$V_2 = j\omega M I_1 + j\omega L_2 I_2$$



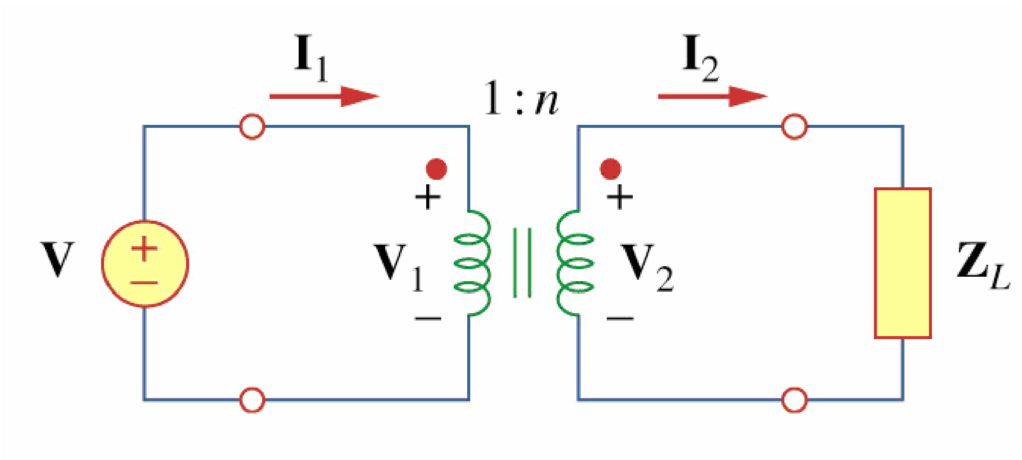
Perfect Coupling $k = 1$, Thus we have $M = \sqrt{L_1 L_2}$

$$V_2 = j\omega L_2 I_2 + \frac{\sqrt{L_1 L_2} V_1}{L_1} - \frac{j\omega L_1 L_2 I_2}{L_1} = \sqrt{\frac{L_2}{L_1}} V_1 = n V_1 = \frac{N_2}{N_1} V_1$$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n = \text{Turns Ratio}$$

Power conservation $\rightarrow$ energy supplied to the primary equals energy absorbed by the secondary.

$$V_1 I_1 = V_2 I_2 \quad \rightarrow \quad \frac{I_1}{I_2} = \frac{V_2}{V_1} = n$$



1. Step-up transformer if n > 1.
2. Step-down transformer if n < 1.
3. Isolation transformer if n=1.

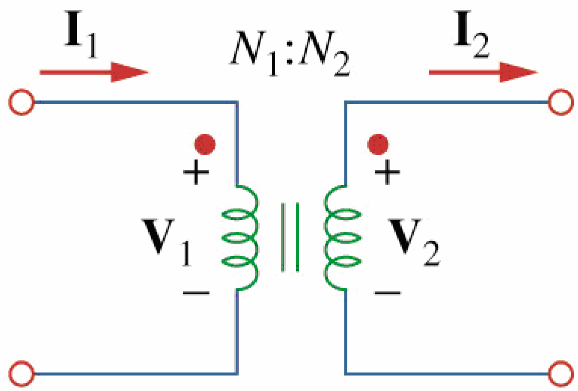
$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = \pm\frac{N_2}{N_1} = \pm n$$

IDEAL TRANSFORMER DOT CONVENTION

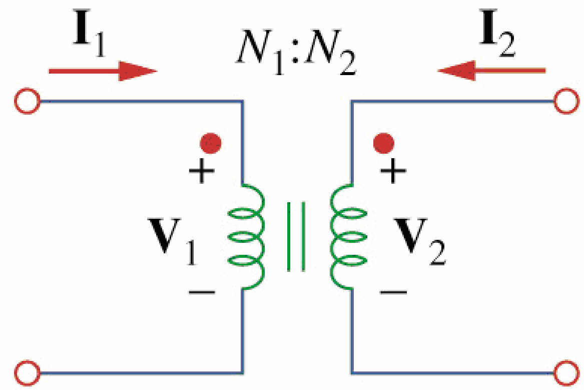
Transformer dot convention is needed to assign the polarity of the output variables.

1) If V_1 and V_2 are both positive or both negative at the dotted terminals, use +n, otherwise –n.

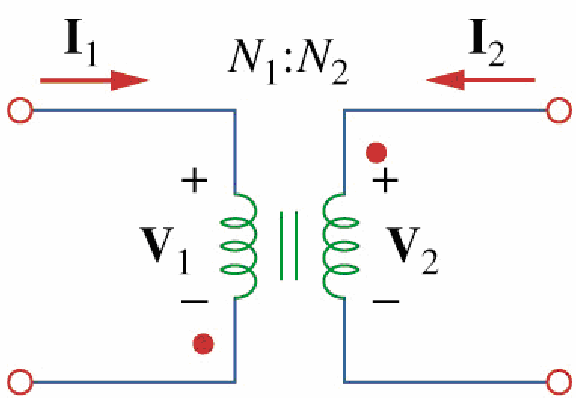
2) If I_1 and I_2 both enter or both leave the dotted terminals, use –n, otherwise +n.



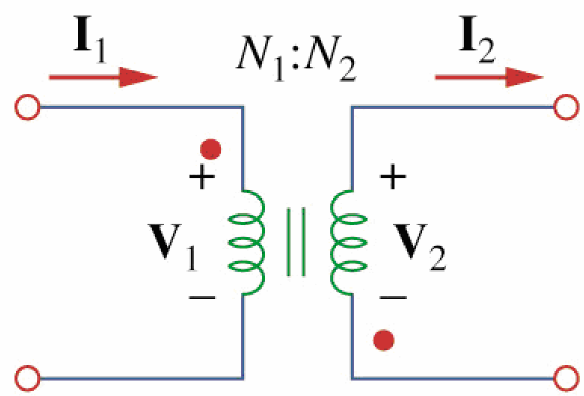
$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2}$$



$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{N_1}{N_2}$$



$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = -\frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2}$$



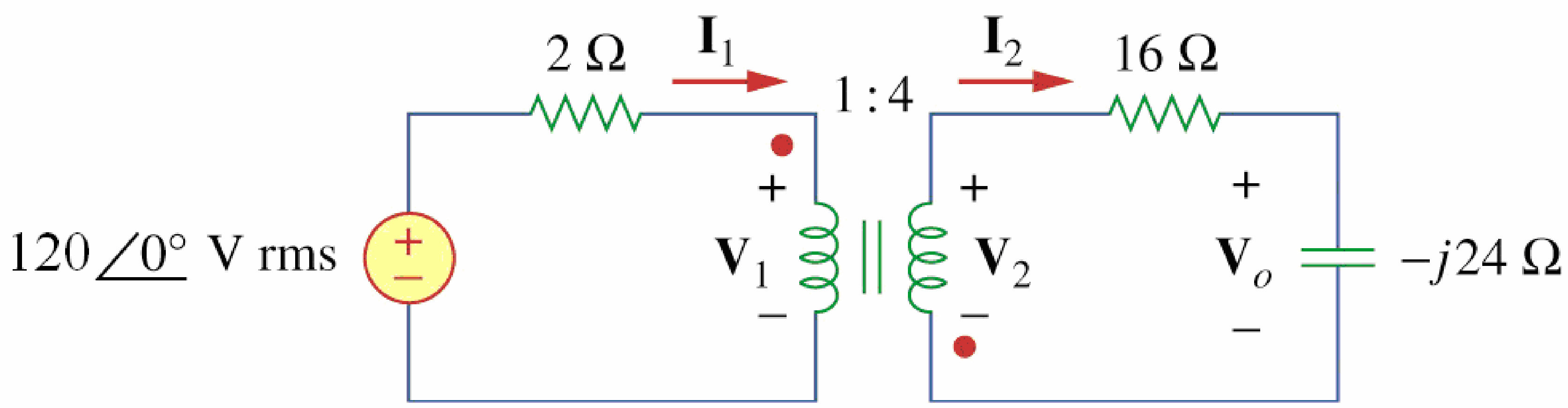
$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = -\frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{N_1}{N_2}$$

Ex. Practice Problem 13.7

The primary current of an ideal transformer rated at 3300/110 V is 5 A. Calculate: (a) the turns ratio, (b) the kVA rating, and (c) the secondary current.

Ex. Practice Problem 13.8

In the ideal transformer circuit shown, find V_0 and the complex power supplied by the source.



Ex. Practice Problem 13.9

Find V_0 in the circuit below.