RESONANT CIRCUITS
- Resonance occurs in a circuit that has at least one inductor and one capacitor.

- Resonance is a condition in an RLC circuit in which the capacitive and inductive reactances are equal in magnitude, resulting in a purely resistive total impedance.

- Can be designed to have a sharp peak in its amplitude characteristic allowing the circuit to discriminate input frequencies.

- Useful for constructing bandpass filters, known as resonators, whose frequency response is highly frequency selective.

- There are two types of resonant circuits: series and parallel resonant circuits.
SERIES RESONANCE

The input impedance is
\[ Z(j \omega) = \frac{V_s(j \omega)}{I(j \omega)} = R + j \left( \omega L - \frac{1}{\omega C} \right) \]

Resonant occurs when \( Z(j \omega) \) is purely real \( \rightarrow \omega L - \frac{1}{\omega C} = 0 \)

The value of \( \omega \) that satisfies such condition is called the resonant frequency:
\[ \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s} \quad \text{or} \quad f_0 = \frac{1}{2\pi \sqrt{LC}} \text{ Hz} \]
FEATURES OF SERIES RESONANCE

1. Transfer impedance, \( H(j\omega) = Z(j\omega) = R \) is purely resistive and its magnitude is minimum.

2. Voltage and current are in phase (unity power factor).

3. The frequency response is measured in terms of the circuit’s current.

\[
|I| = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}
\]

4. The average power dissipated by the circuit is \( P(\omega) = \frac{1}{2} |I|^2 R \)

5. The average power reaches its maximum at resonance \( (\omega = \omega_0) \)

where \( |I| = \frac{V_m}{R} \) and \( P(\omega_0) = \frac{1}{2} \frac{V_m^2}{R} \)
6. The half-power frequencies are obtained by

\[ P(\omega) = \frac{1}{2} |I|^2 R = \frac{1}{2} \left( \frac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \right)^2 \]

\[ R = \frac{1}{2} P(\omega_0) = \frac{1}{2} \left( \frac{1}{2} \frac{V_m^2}{R} \right) \]

\[ R^2 + \left( \omega L - \frac{1}{\omega C} \right)^2 = 2R^2 \]

\[ \omega_1 = -\frac{R}{2L} + \sqrt{\left( \frac{R}{2L} \right)^2 + \frac{1}{LC}} \]

\[ \omega_2 = \frac{R}{2L} + \sqrt{\left( \frac{R}{2L} \right)^2 + \frac{1}{LC}} \]

\[ P(\omega_1) = P(\omega_2) = \frac{(V_m / \sqrt{2})^2}{2R} = \frac{V_m^2}{4R} \]

\[ \omega_0 = \sqrt{\omega_1 \omega_2} \]
7. The width of the response curve is called the **half-power bandwidth** and defined as the difference between the two half-power frequencies:

\[ B = \omega_2 - \omega_1 = \frac{R}{L} = \omega_0^2 R C \]

8. The height of the curve depends on the value of \( R \)

9. If \( R \) is made smaller with a fixed \( L \) and \( C \), the bandwidth decreases and the selectivity increases.

10. If the ratio \( L/C \) increases with fixed \( R \), the bandwidth decreases with an increase in selectivity.
10. The circuit’s selectivity is the ability to respond to a certain frequency and discriminate against other frequencies.

11. It refers to the “sharpness” of the resonance which depends on the height and the width of the response curve.

12. Quantitatively, selectivity is measured by the quality factor $Q$.

$$Q = 2\pi \frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated in one period at resonance}}$$

$$Q = 2\pi \frac{\frac{1}{2}LI^2}{\frac{1}{2}I^2R\left(\frac{1}{f_0}\right)} = \frac{2\pi f_0L}{R}$$

$$Q = \frac{\omega_0L}{R} = \frac{1}{\omega_0RC} \quad \rightarrow \quad B = \frac{R}{L} = \frac{\omega_0}{Q}$$
13. **QUALITY FACTOR** is the ratio of the resonance frequency to the bandwidth. The higher the Q the smaller the bandwidth.

14. For high-Q circuits \((Q \geq 10)\), \(\omega_1\) and \(\omega_2\) are symmetrical around \(\omega_0\).

\[
\omega_1 \approx \omega_0 - \frac{B}{2}
\]

\[
\omega_2 \approx \omega_0 + \frac{B}{2}
\]
$V_{C_{\text{max}}} = V_{L_{\text{max}}}$

$V_{C_s} = V_{L_s} = QE$
EQ15 GRAPHIC EQUALIZER

CHANNEL A

CHANNEL B

Dual Channel Constant Q Graphic Equalizer

Volume

f (log scale)

10 Hz 100 Hz 1 kHz 10 kHz 100 kHz

31 Hz 63 Hz 125 Hz 250 Hz 500 Hz 2 kHz 4 kHz 8 kHz 16 kHz
Ex. (Practice problem 14.7)
A series-connected circuit has $R = 4\Omega$ and $L = 25$ mH.
(a) Calculate the value of $C$ that will produce a quality factor of 50.
(b) Find the half-power frequencies and the bandwidth
(c) Determine the average power dissipated at $\omega_0, \omega_1, \omega_2$. 
Take $V_m=100$ V.
The input admittance is

\[ Y(j\omega) = \frac{I(j\omega)}{V_s(j\omega)} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) \]

Resonant occurs when \( Y(j\omega) \) is purely real \( \rightarrow \omega L - \frac{1}{\omega C} = 0 \)

The value of \( \omega \) that satisfies such condition is called the resonant frequency:

\[ \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s} \quad \text{or} \quad f_0 = \frac{1}{2\pi \sqrt{LC}} \text{ Hz} \]
The frequency response is measured in terms of the circuit’s common voltage.

\[
|V| = \frac{I_m}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}}
\]

\[
\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}
\]

\[
\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}
\]

\[
B = \omega_2 - \omega_1 = \frac{1}{RC} = \omega_0^2 \frac{L}{R}
\]

\[
Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}
\]

For high-Q circuits, \(\omega_1 \approx \omega_0 - \frac{B}{2}\) and \(\omega_2 \approx \omega_0 + \frac{B}{2}\)
For both the series and parallel resonant circuits, the half-power frequencies can be expressed in terms of $Q$ as

$$\omega_1 = \omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} - \frac{\omega_0}{2Q}$$

$$\omega_1 = \omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} + \frac{\omega_0}{2Q}$$
Ex. Practice Problem 14.8
A parallel resonant circuit has $R = 100 \, \text{k}\Omega$, $L = 20 \, \text{mH}$, and $C = 5 \, \text{nF}$. Calculate $\omega_0, \omega_1, \omega_2, Q, B$. 
Ex. Practice Problem 14.9
Calculate the resonant frequency of the circuit shown.