Homework #2

|1| (a) A Si sample is doped with $10^{17}$ boron atoms / cm$^3$. What is the electron concentration $n_0$ at 300 K? What is the resistivity when $\mu_n = 900$ cm$^2$ / V-s and $\mu_p = 320$ cm$^2$ / V-s at this doping concentration?

(b) A Ge sample is doped with $3 \times 10^{13}$ Sb atoms / cm$^3$. Using the requirements of space charge neutrality, calculate the electron concentration $n_0$ at 300 °K.

(Streetman, 3rd.Ed., 3.9)

(a) \textbf{Given:}
- $N_A = 10^{17}$ atoms / cm$^3$
- $T = 300$ °K
- $\mu_n = 900$ cm$^2$ / V-s
- $\mu_p = 320$ cm$^2$ / V-s

\textbf{Find:}
- $n_0 = ?$
- $\rho = ?$

\textbf{Solution:}

$$p_0 \approx N_A = 10^{17}$$

The intrinsic carrier concentration for Si is $n_i = 1.5 \times 10^{10}$ cm$^{-3}$.

$$n_0 p_0 \approx n_i^2 = (1.5 \times 10^{10})^2$$

$$n_0 \approx \frac{n_i^2}{p_0} = \frac{(1.5 \times 10^{10})^2}{10^{17}}$$
Answer: \( n_0 = 2.25 \times 10^3 \text{ cm}^3 \)

The conductivity of the sample is,

\[
\sigma = q \left( \mu_n n_0 + \mu_p p_0 \right) = (1.6 \times 10^{-19})[900(2.25 \times 10^4) + 320(10^{17})]
\]

\[
\sigma = 5.12 \ (\Omega \text{-cm})^{-1}
\]

\[
\rho = \frac{1}{\sigma} = \frac{1}{5.12}
\]

Answer: \( \rho = 0.195 \ \Omega \text{-cm} \)

(b) Given: \( N_D = 3 \times 10^{13} \text{ cm}^3 \) Germanium sample
\( T = 300 \ ^\circ\text{K} \)

Find: \( n_0 =? \)

Solution:

The doping concentration is close to the intrinsic carrier concentration for Ge, use charge neutrality ,

\[
p_0 + N_D^- = n_0 + N_A^-
\]

where \( N_A = 0 \), so that,

\[
n_0 = N_D^+ + p_0
\]

and,

\( \cdots = \cdots^2 \)
substituting for \( p_0 \).

\[
p_0 = \frac{n_i^2}{n_0}
\]

so that,

\[
n_0 = N_D^+ + \frac{n_i^2}{n_0}
\]

\[
n_0^2 - N_D^+ n_0 - n_i^2 = 0
\]

\[
n_0 = \frac{N_D \pm \sqrt{N_D^2 + 4n_i^2}}{2} = \frac{(3 \times 10^{13}) \pm \sqrt{(3 \times 10^{13})^2 + 4(2.5 \times 10^{13})^2}}{2}
\]

Answer: \( n_0 = 4.42 \times 10^{13} \text{ cm}^{-3} \)
\[ N_D = 1 \times 10^{13} \text{ cm}^3 \text{ and } N_A = 2.5 \times 10^{13} \text{ cm}^3 \text{ at } T = 300 \degree \text{K} \text{. For each of the three materials (a) Is this material n-type or p-type? (b) Calculate } n_0 \text{ and } p_0 \text{.} \]

(Neamen, 3.24)

\[ \text{Given:} \quad \text{Si, Ge, and GaAs material} \quad T = 300 \degree \text{K} \]

\[ N_D = 1 \times 10^{13} \text{ cm}^3 \]

\[ N_A = 2.5 \times 10^{13} \text{ cm}^3 \]

(a) \text{ Find: } n- \text{ or p-type} = ?

\text{Solution:}

Since \( N_A > N_D \),

\[ \text{Answer: all of the materials are doped p-type} \]

(b) \text{ Find: } n_0 = ? \quad p_0 = ?

\text{Solution:}

Charge neutrality requires that,

\[ n_0 + N_A^- = p_0 + N_D^+ \]

and

\[ n_0 p_0 = n_i^2 \]

\[ \frac{n_i^2}{p_0} + N_A^- = p_0 + N_D^- \]
\[p_0^2 + (N_D - N_A) p_0 - n_i^2 = 0\]

\[p_0 = \frac{-(N_D - N_A) \pm \sqrt{(N_D - N_A)^2 + 4n_i^2}}{2}\]

where,

\[(N_D - N_A) = (1 \times 10^{13}) - (2.5 \times 10^{13}) = -1.5 \times 10^{13}\]

For Si, \(n_i = 1.5 \times 10^{10}\) cm\(^{-3}\)

\[p_0 = \frac{1.5 \times 10^{13} \pm \sqrt{(1.5 \times 10^{13})^2 + 4(1.5 \times 10^{10})^2}}{2}\]

\[\text{Answer: } p_0 = 1.5 \times 10^{13}\) cm\(^{-3}\)

\[n_0 = \frac{n_i^2}{p_0} = \frac{(1.5 \times 10^{10})^2}{1.5 \times 10^{13}}\]

\[\text{Answer: } n_0 = 1.5 \times 10^7\) cm\(^{-3}\)

For Ge, \(n_i = 2.4 \times 10^{13}\) cm\(^{-3}\)

\[p_0 = \frac{1.5 \times 10^{13} \pm \sqrt{(1.5 \times 10^{13})^2 + 4(2.4 \times 10^{13})^2}}{2}\]
\[ n_0 = \frac{n_i^2}{p_0} = \frac{(2.4 \times 10^{13})^2}{3.26 \times 10^{13}} \]

Answer: \[ p_0 = 3.26 \times 10^{13} \text{ cm}^{-3} \]

\[ n_0 = \frac{n_i^2}{p_0} = \frac{(1.8 \times 10^{610})^2}{1.5 \times 10^{13}} \]

Answer: \[ n_0 = 0.22 \text{ cm}^{-3} \]

Note: This material is just slightly p-type, it is almost intrinsic.

For GaAs, \[ n_i = 1.8 \times 10^6 \text{ cm}^{-3} \]

\[ p_0 = \frac{1.5 \times 10^{13} \pm \sqrt{(1.5 \times 10^{13})^2 + 4(1.8 \times 10^{610})^2}}{2} \]

Answer: \[ p_0 = 1.5 \times 10^{13} \text{ cm}^{-3} \]
For a particular semiconductor material, $T = 300 \text{ K}$, $N_C = 1 \times 10^{18} \text{ cm}^{-3}$, and $N_V = 10^{19} \text{ cm}^{-3}$. Let $E_g = 1.45 \text{ eV}$. Determine the position of the intrinsic Fermi level with respect to the center of the bandgap. (Neamen, 3.8)

Given:

$E_g = 1.45 \text{ eV}$

$N_C = 1 \times 10^{18} \text{ cm}^{-3}$

$N_V = 10^{19} \text{ cm}^{-3}$

Find:

$E_i - E_{\text{midgap}}$?

Solution:

For an intrinsic material, the electron concentration and the hole concentration are equal.

$n_0 = p_0$

$n_0 = N_C \exp \left[ -\frac{(E_C - E_i)}{kT} \right] = N_V \exp \left[ -\frac{(E_i - E_V)}{kT} \right] = p_0$

$\exp \left[ -\frac{(E_C - E_i)}{kT} \right] = \frac{N_V}{N_C}$

$\exp \left[ -\frac{(E_i - E_V)}{kT} \right] = \frac{N_V}{N_C}$

$\exp \left[ -\frac{(E_C - E_i)}{kT} \right] \exp \left[ \frac{(E_i - E_V)}{kT} \right] = \frac{N_V}{N_C}$

$\exp \left[ \frac{(-E_C + E_i + E_i - E_V)}{kT} \right] = \frac{N_V}{N_C}$
\[ \exp \left[ \frac{2E_i - (E_C + E_V)}{kT} \right] = \frac{N_V}{N_C} \]

\[ 2E_i - (E_C + E_V) = kT \ln \left( \frac{N_V}{N_C} \right) \]

\[ E_i - \frac{(E_C + E_V)}{2} = \frac{kT}{2} \ln \left( \frac{N_V}{N_C} \right) \]

\[ E_i - E_{midgap} = \frac{kT}{2} \ln \left( \frac{N_V}{N_C} \right) = \frac{0.026}{2} \ln \left( \frac{1 \times 10^{19}}{1 \times 10^{18}} \right) \]

\textit{Answer:} \quad E_i - E_{midgap} = 0.03 \text{ eV}
The Fermi energy for copper at $T = 300 \text{ K}$ is 7.0 eV. The electrons in copper follow the Fermi-Dirac distribution function. (a) Find the probability of an energy level at 7.15 eV being occupied by an electron. (b) Repeat part (a) for $T = 1000 \text{ K}$. (Assume that $E_F$ is a constant.) (c) Repeat part (a) for $E = 6.85$ eV and $T = 300 \text{ K}$. (d) Determine the probability of the energy state at $E = E_F$ being occupied at $T = 300 \text{ K}$ and at $T = 1000 \text{ K}$. (Neamen, 2.28)

Given:  

$E_F = 7.0 \text{ eV} \quad \text{ at } T = 300 \text{ K}$

Fermi-Dirac distribution applies

(a) Find: $f(E) = ?$ for an electron with $E = 7.15 \text{ eV}$

Solution:

The Fermi-Dirac distribution function is given by,

$$f(E) = \frac{1}{1 + \exp \left( \frac{E - E_F}{kT} \right)}$$

Then, for an electron with $E = 7.15 \text{ eV},$

$$f(E) = \frac{1}{1 + \exp \left( \frac{7.15 - 7.0}{0.0259} \right)}$$

(a) Answer: $f(E) = 3.05 \times 10^{-3}$

(b) Find: $f(E) = ?$ for an electron with $E = 7.15 \text{ eV}$ at $T = 1000 \text{ K}$

Solution:

$$kT = (8.62 \times 10^{-5})(1000) = 0.0862 \text{ eV}$$
\[ f(E) = \frac{1}{1 + \exp \left( \frac{0.15}{0.0862} \right)} \]

(b) Answer: \( f(E) = 0.149 \)

(c) Find: \( f(E) = ? \) if \( E = 6.85 \text{ eV} \) at \( T = 300 \, \text{°K} \)

\[ f(E) = \frac{1}{1 + \exp \left( \frac{-0.15}{0.0259} \right)} \]

(c) Answer: \( f(E) = 0.997 \)

(d) Find: \( f(E) = ? \) for electrons at \( E = E_F \) at \( T = 300 \, \text{°K} , \ T = 1000 \, \text{°K} \)

\[ f(E) = \frac{1}{1 + \exp \left( \frac{0}{kT} \right)} \]

(d) Answer: \( f(E) = 0.5 \) independent of the temperature
A silicon sample at 300 K is doped with $3 \times 10^{15}$ cm$^{-3}$ acceptor atoms. Find the donor atom concentration that must be added to the sample so that the electron concentration is $n_0 = 10^{16}$ cm$^{-3}$. What is the position of the Fermi energy level with respect to the conduction band edge, that is $E_C - E_F$? You may assume that the intrinsic Fermi energy level is at midgap.

**Given:**
- silicon sample $T = 300\,^\circ$K
- $E_i$ is at midgap
- $N_A = 3 \times 10^{15}$ cm$^{-3}$

**Find:**
- $N_D = ?$ so that $n_0 = 10^{16}$ cm$^{-3}$
- $E_C - E_F = ?$

**Solution:**

When $n_0 = 10^{16}$ cm$^{-3}$, the hole concentration is,

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \, \text{cm}^{-3}$$

Use the equation for charge neutrality,

$$n_0 + N_A^- = p_0 + N_D^+$$

All of the dopant atoms are ionized at 300 $^\circ$K, so that,

$$N_D^+ = n_0 + N_A^- - p_0 = 10^{16} + 3 \times 10^{15} - 2.25 \times 10^4$$

**Answer:**

$$N_0 = 1.3 \times 10^{16} \, \text{cm}^{-3}$$

To find the position of the Fermi level,

$$n_0 = n_i \, e^{(E_F - E_i) / kT}$$
\[ E_F - E_i = kT \ln \left( \frac{n_0}{n_i} \right) = 0.0259 \ln \left( \frac{10^{16}}{1.5 \times 10^{10}} \right) \]

\[ E_F - E_i = 0.347 \text{ eV} \]

\[ E_C - E_F = (E_C - E_i) - (E_F - E_i) = 0.56 - 0.347 \]

*Answer:* \( E_C - E_F = 0.213 \text{ eV} \)