HW 5 Solutions

Problem 1: D.1 (10 points) at the end of Module D in your text.

D.1 This is an $M/M/1$ queue; λ = 3/hr; and μ = 5/hr.

(a) $L_q = \dfrac{\lambda^2}{\mu(\mu-\lambda)} = \dfrac{3^2}{5(5-3)} = \dfrac{9}{5(2)} = 0.9$ persons

(b) $L_s = \dfrac{\lambda}{\mu-\lambda} = \dfrac{3}{5-3} = \dfrac{3}{2} = 1.5$ persons

(c) $W_q = \dfrac{\lambda}{\mu(\mu-\lambda)} = \dfrac{3}{5(5-3)} = \dfrac{3}{10}\text{hr} = 18\ \text{min}$

(d) $W_s = \dfrac{1}{\mu-\lambda} = \dfrac{1}{5-3} = \dfrac{1}{2}\text{hr} = 30\ \text{min}$

(e) $\rho = \dfrac{\lambda}{\mu} = \dfrac{3}{5} = 0.60$, or 60%

Problem 2: D.7 (10 points) at the end of Module D in your text.

D.7 This is a single server with unlimited potential customer population so we use the $M/M/1$ model.
λ = 24 cars per hour (that's 4 every ten minutes), and μ = 30 cars per hour (that's one every two minutes).

(a) $W_s = \dfrac{1}{\mu - \lambda} = \dfrac{1}{30 - 24} = \dfrac{1}{6}$ hours

(b) $L_s = \dfrac{\lambda}{\mu - \lambda} = \dfrac{24}{30 - 24} = \dfrac{24}{6} = 4$ cars

(c) $L_q = \dfrac{\lambda^2}{\mu(\mu - \lambda)} = \dfrac{24^2}{30(30 - 24)} = \dfrac{24^2}{30(6)} = 3.2$ cars

(d) $W_q = \dfrac{\lambda}{\mu(\mu - \lambda)} = \dfrac{24}{30(30 - 24)} = \dfrac{24}{30(6)} = \dfrac{2}{15}$ hours

(e) $P_0 = 1 - \lambda/\mu = 1 - 24/30 = \dfrac{1}{5}$

(f) $\rho = \dfrac{\lambda}{\mu} = \dfrac{24}{30} = \dfrac{4}{5}$

(g) Probability(n = 2) = $P_{n>1} - P_{n>2}$

$$= \left(\frac{24}{30}\right)^{1+1} - \left(\frac{24}{30}\right)^{2+1} = 0.128$$

Problem 3: D.9 (10 points) at the end of Module D in your text.

D.9 This is an *M/M/*1 model; $\lambda = 10$, $\mu = 15$

(a) $W_q = \dfrac{\lambda}{\mu(\mu - \lambda)} = \dfrac{10}{15(15-10)}$

$= \dfrac{10}{15(5)} = \dfrac{2}{15} = 0.1333$ hours

(b) $L_q = \dfrac{\lambda^2}{\mu(\mu - \lambda)} = \dfrac{10^2}{15(15-10)} = \dfrac{10^2}{15(5)} = 1.333$

(c) $W_s = \dfrac{1}{\mu - \lambda} = \dfrac{1}{15-10} = \dfrac{1}{5}$ hours

(d) $L_s = \dfrac{\lambda}{\mu - \lambda} = \dfrac{10}{15-10} = \dfrac{10}{5} = 2$

(e) $P_0 = 1 - \lambda/\mu = 1 - 10/15 = 1/3$

(f) This is an *M/M/*2 model; $\lambda = 10$, $\mu = 15$

(a) $W_q = 0.0083$ hours
(b) $L_q = 0.0833$
(c) $W_s = 0.075$ hours
(d) $L_s = 0.75$
(e) $P_0 = 0.5$

Problem 4: D.13 (10 points) at the end of Module D in your text.

D.13 $\lambda = 12$ calls/hour, $\mu = 60/4 = 15$ calls/hour

(a) The average time the catalogue customer must wait, W_q, is given by:

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{12}{15(15-12)} = \frac{12}{15 \times 3}$$
$$= \frac{12}{45} = 0.267 \text{ hours}$$

(b) The average number of callers waiting to place an order, L_q, is given by:

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{12^2}{15(15-12)} = \frac{144}{15 \times 3}$$
$$= \frac{144}{45} = 3.2 \text{ customers}$$

(c) To decide whether or not to add the second clerk, we must

- Compute present total cost
- Compute total cost with the second clerk
- Compare the two

Present total cost, on an hourly basis:

C_t / hour = service cost + waiting cost

= \$10 per hour +
(12 calls per hour × 0.267 hours
waiting per call × \$25 per hour)

$= 10 + (12 \times 0.267 \times 25)$

$= 10 + 80.1/\text{hour}$

$= \$90.10/\text{hour}$

To determine total cost using the second clerk (a second channel):

$$P_0 = \frac{1}{\left[\sum_{n=0}^{n=M-1} \frac{1}{n!}\left[\frac{\lambda}{\mu}\right]^n\right] + \frac{1}{M!}\left[\frac{\lambda}{\mu}\right]^M \frac{M\mu}{M\mu-\lambda}}$$

$$= \frac{1}{\frac{1}{0!}\left[\frac{12}{15}\right]^0 + \frac{1}{1}\left[\frac{12}{15}\right]^1 + \frac{1}{1\times 2}\left[\frac{12}{15}\right]^2 \frac{2\times 15}{2\times 15-12}}$$

$$= \frac{1}{1+\frac{4}{5}+\frac{1}{2}\left[\frac{4}{5}\right]^2 \frac{2\times 15}{30-12}}$$

$$= \frac{1}{1+\frac{4}{5}+\left(\frac{1}{2}\right)\left(\frac{16}{25}\right)\left(\frac{30}{18}\right)}$$

$$= \frac{1}{1+\frac{4}{5}+\frac{480}{900}}$$

or:

$$P_0 = \frac{1}{1+0.8+0.53} = \frac{1}{2.33} = 0.429$$

$$W_q = \frac{\mu\left(\frac{\lambda}{\mu}\right)^M}{(M-1)!(M\mu-\lambda)^2} P_0$$

Then:

$$W_q = \frac{15\times\left(\frac{12}{15}\right)^2}{(2-1)!(2\times 15-12)^2}\times 0.429$$

$$= \frac{15\times 0.64}{1\times(30-12)^2}\times 0.429$$

$$= \frac{4.12}{1\times 324} = 0.0127 \text{ hours}$$

$$= 0.763 \text{ minutes}$$

Cost with two clerks:

C_t/ hour = service cost + waiting cost

$$= 20 + 12\frac{\text{calls}}{\text{hour}}\times 0.0127\frac{\text{hours}}{\text{call}}\times 25\frac{\$}{\text{hour}}$$

$$= 20 + 12\times 0.0127\times 25$$

$$= 20 + 3.81 = \$23.81/\text{ hour}$$

There is a saving of $90.10 –$23.81 = $66.29/hour

Thus, a second clerk should certainly be added!

(d) (Optional – not asked) Would 3 clerks be better? With three clerks the cost goes to $30.47. So the costs are:

1 clerk	$90.10
2 clerks	$23.81
3 clerks	$30.47

For 3 clerks W_q = 0.00158 (from Excel OM)

$30 + (12 × 0.0015 × $25) = $30 + 0.47 = $30.47

Therefore, optimum number of clerks is 2.

Problem 5: D.19 (10 points) at the end of Module D in your text.

D.19 First, the cumbersome way, according to the text. Then the easy way, using software.

The cumbersome way:

$N = 5$ drilling machines, $M = 1$ mechanic, $T = 1$ day, $U = 6$ days

$$X = \frac{T}{T+U} = \frac{1}{1+6} = 0.143$$

The value 0.145 will be used for X when referencing Table D.7.

(a) The average number of machines waiting for service, L_q, is given by:

$$L_q = N(1 - F)$$

where F is found from Table D.7. From Table D.7, when $M = 1$, $X = 0.145$; $F = 0.892$

$$L_q = 5(1 - 0.892) = 0.54 \text{ machines waiting}$$

(b) The average number of machines in running order, J, is given by: $J = NF(1 - X)$ where F is found from Table D.7. From Table D.7, with $M = 1$, $X = 0.145$; $F = 0.892$

$$J = 5 \times 0.892 \times (1 - 0.145) = 3.81 \text{ machines}$$

(c) The reduction in waiting time obtained by employing a second mechanic is found as follows: Waiting time employing a single mechanic, W_1, is given by:

$$W_1 = \frac{T(1-F)}{XF}$$

where F is found from Table D.7. From Table D.7, when $M = 1$, $X = 0.145$; $F = 0.892$.

$$W_1 = \frac{1 \times 1(1 - 0.892)}{0.145 \times 0.892} = 0.835 \text{ days}$$

From Table D.7, with $M = 2$, $X = 0.145$: $F = 0.991$

$$W_2 = \frac{1 \times 1(1 - 0.991)}{0.145 \times 0.991} = 0.063 \text{ days}$$

The time saved is given by: $W_1 - W_2$

Time saved = 0.835 – 0.063 = 0.772 days

The easy Way: (Using software)

a. Lq = 0.5228 machines
b. Avg # drills in running order = 5 - 1.16 = 3.84
c. A second mechanic would reduce mean waiting time by 0.7559 days

Waiting times (days):

	One Mechanic	Two Mechanics	Difference
Average waiting/service time in the system (W)	1.8174	1.0615	0.7559
Average waiting time in line (W_q)	0.8174	0.0615	0.7559

Problem 6: 12.7 (10 points) at the end of Chapter 12 in your text.

12.7 This problem reverses the unknown of a standard EOQ problem to solve for S.

$$60 = \sqrt{\frac{2 \times 240 \times S}{.4 \times 10}}\ ;\ \text{or } 60 = \sqrt{\frac{480S}{4}},\ \text{or}$$

$$60 = \sqrt{120S}\text{, so solving for } S \text{ results in } S = \frac{3{,}600}{12} = \$30.$$

That is, if S were $30, then the EOQ would be 60. If the true ordering cost turns out to be much greater than $30, then the firm's order policy is ordering too little at a time.

Problem 7: 12.9 (10 points) at the end of Chapter 12 in your text.

12.9 $D = 15{,}000$, $H = \$25\text{/unit/year}$, $S = \$75$

(a) $\text{EOQ} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 15{,}000 \times 75}{25}} = 300 \text{ units}$

(b) Annual holding costs $= (Q/2) \times H = (300/2) \times 25 = \$3{,}750$

(c) Annual ordering costs $= (D/Q) \times S = (15{,}000/300) \times 75 = \$3{,}750$

(d) $\text{ROP} = d \times L = \left(\frac{15{,}000 \text{ units}}{300 \text{ days}}\right) \times 2 \text{ days} = 100 \text{ units}$

Problem 8: 12.13 (10 points) at the end of Chapter 12 in your text.

12.13 (a) $Q = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(2500)18.75}{1.50}}$

$= 250$ brackets per order

(b) Average inventory $= \frac{Q}{2} = \frac{250}{2} = 125$ units

Annual holding cost $= \frac{Q}{2}H = 125(1.50) = \$187.50$

(c) Number of orders $= \frac{D}{Q} = \frac{2500}{250} = 10$ orders/year

Annual order cost $= \frac{D}{Q}S = 10(18.75) = \$187.50$

(d) $\text{TC} = \frac{Q}{2}H + \frac{D}{Q}S = 187.50 + 187.50 = \$375/\text{year}$

(e) Time between orders $= \frac{\text{working days}}{(D/Q)}$

$= \frac{250}{10} = 25$ days

(f) $\text{ROP} = dL = 10(2) = 20$ units (where $10 =$ daily demand)

$d = \frac{2500}{250} = 10$

Problem 9: 12.21 (5 points) at the end of Chapter 12 in your text.

12.21 The solution to any quantity discount model involves determining the total cost of each alternative after quantities have been computed and adjusted for the original problem and every discount.

We start the analysis with no discount:

$$\text{EOQ (no discount)} = \sqrt{\frac{2(1{,}400)(25)}{0.2(400)}}$$

$$= 29.6 \text{ units}$$

$$\text{Total cost (no discount)} = \text{Cost of goods} + \text{Ordering cost} + \text{Carrying cost}$$

$$= \$400(1{,}400) + \frac{1{,}400(25)}{29.6} + \frac{29.6(\$400)(0.2)}{2}$$

$$= \$560{,}000 + \$1{,}183 + \$1{,}183$$

$$= \$562{,}366$$

The next step is to compute the total cost for the discount:

$$\text{EOQ (with discount)} = \sqrt{\frac{2(1{,}400)(25)}{0.2(\$380)}}$$

$$= 30.3 \text{ units}$$

$$\text{EOQ (adjusted)} = 300 \text{ units}$$

Because this last economic order quantity is below the discounted price, we must adjust the order quantity to 300 units. The next step is to compute total cost.

$$\text{Total cost (with discount)} = \text{Cost of goods} + \text{Ordering cost} + \text{Carrying cost}$$

$$= \$380(1{,}400) + \frac{1{,}400(25)}{300} + \frac{300(\$380)(0.2)}{2}$$

$$= \$532{,}000 + \$117 + \$11{,}400$$

$$= \$543{,}517$$

The optimal strategy is to order 300 units at a total cost of \$543,517.

Problem 10: 12.27 (5 points) at the end of Chapter 12 in your text.

12.27 (a) $\mu = 60$; $\sigma = 7$

Safety stock for 90% service level = $\sigma Z(\text{at } 0.90)$
$= 7 \times 1.28 = 8.96 \approx 9$

(b) ROP = 60 + 9 = 69 BX-5 bandages.

Problem 11: 12.31 (5 points) at the end of Chapter 12 in your text.

12.31

If Mr. Beautiful carries added safety stock it will cost $5 per year for each unit as additional carrying cost. Stockout costs will go down based on the probabilities given, the $50 per set stockout cost, and the 7 times an order is placed per year. In the table below notice the total cost keeps going down as we add safety stock. All demand is met with safety stock of 30 units (total stock of 90 units for reorder point).

Safety Stock	Additional Carrying Cost	Stockout Cost	Total Cost
0	0	$10 \times 0.2 \times 50 \times 7 + 20 \times 0.2 \times 50 \times 7 + 30 \times 0.1 \times 50 \times 7 = 3{,}150$	3,150
10	$10 \times 5 = 50$	$50 \times 7(10 \times 0.2 + 20 \times 0.1) = 1{,}400$	1,450
20	$20 \times 5 = 100$	$10 \times 0.1 \times 50 \times 7 = 350$	450
30	$30 \times 5 = 150$	0	150

The BB-1 set should therefore have a safety stock of 30 units; ROP = 90 units.

Problem 12: (5 points)

Problem 12 - You sell on average 300 units of a certain product each day. Sales follow a normal distribution and the standard deviation of sales for this product is 24 units. This product is supplied to you from another state and the lead time for the delivery is 10 days with a standard deviation of 2 days. If you want to maintain a 95% service level what would be your reorder point?

Both demand and lead time are variable. Note this problem is worked (with different numbers) on page 507 of your text.

ROP = (average daily demand x average lead time) + $Z\sigma_{dLT}$

σ_{dLT} = SqRoot { (average lead time x σ_d^2) + (average daily demand)^2 x $(\sigma_{LT})^2$ }

where σ_d = standard deviation of demand per day and σ_{LT} = standard deviation of lead time in days, and Z is the number of standard deviations corresponding to a desired service level.

Given:

Average daily demand = 300 units, with standard deviation of 24 units (normally distributed).
Average lead time = 10 days, with standard deviation of 2 days (normally distributed).
Service level = 95%, so the corresponding Z value = 1.645 standard deviations.

σ_{dLT} = SqRoot { $(10 \times 24^2) + (300)^2 \times (2)^2$ }

σ_{dLT} = SqRoot { 5760 + 360000 } = SqRoot {365760} = 604.8

ROP = (300 x 10) + (1.645)(604.8) = **3995 units** ← **Answer**